Transformation Stretch Solutions

October 6, 2004

Problem 1

A shape is translated so that the point $(7, 2)$ moves to $(15, 15)$. Under the same translation, to what point does the point $(-8, 3)$ move?

In a translation, the new coordinates $(x_{\text{new}}, y_{\text{new}})$ are related to the old coordinates $(x_{\text{old}}, y_{\text{old}})$ via a transformation of the form

$$x_{\text{new}} = x_{\text{old}} + \Delta x, \quad y_{\text{new}} = y_{\text{old}} + \Delta y$$

We can easily solve these for Δx and Δy using the given data:

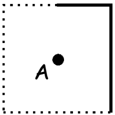
$$\Delta x = x_{\text{new}} - x_{\text{old}} = 15 - 7 = 8, \quad \Delta y = y_{\text{new}} - y_{\text{old}} = 15 - 2 = 13.$$

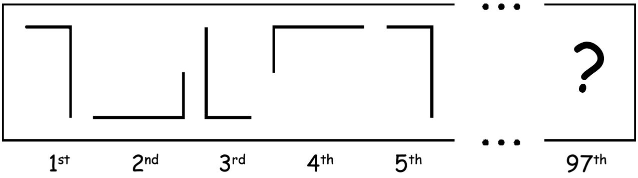
Applying these to the point $(-8, 3)$ yields

$$(x_{\text{new}}, y_{\text{new}}) = (-8 + \Delta x, 3 + \Delta y) = (-8 + 8, 3 + 13) = \boxed{(0, 16)}$$

Problem 2

Yusuf is creating a border pattern for a wall. Using the shape shown to the right, the first five shapes of the pattern are given below. Each time, Yusuf moves the shape to the right and rotates it 90° clockwise about point A. Draw the 97th shape of the pattern in the answer blank.





Note that the patterns repeat with a period of 4, e.g., Pattern 1 = Pattern 5 = Pattern 9 = $\cdots$ Since $97 = 1 + 4 \times 24$ then Pattern 97 will be identical to Pattern 1.

Problem 3

Triangle ABC has vertices $A(2, 1)$, $B(3, 0)$ and $C(4, 2)$. The triangle is reflected over the line $y = x$ to form triangle $A'B'C'$. What is the sum of the x-coordinates of the vertices of triangle $A'B'C'$?

$$\text{Reflect through line } x = y \iff \text{Swap } x \text{ and } y \text{ coordinates}$$

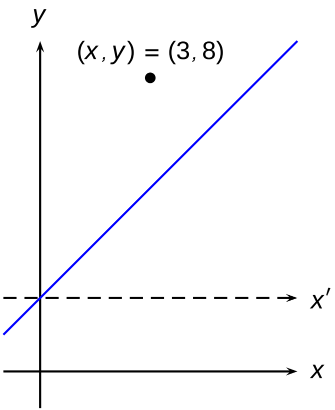
Since the x-coordinates of $A'B'C'$ are the same as the y-coordinates of ABC, the desired sum is just

$$1 + 0 + 2 = \boxed{3}$$

Problem 4

A shape that includes the point $(3, 8)$ is reflected across the line $y = x + 2$. At what point does the point $(3, 8)$ land?

Suppose we move the x-axis up 2 units to form the x'-axis. In the new primed coordinate system, the line $y = x + 2$ becomes $y' = x'$. The point $(x, y) = (3, 8)$ becomes $(x', y') = (3, 6)$ and its reflection through the line $y' = x'$ has coordinates $(x', y') = (6, 3)$ or $(x, y) = \boxed{(6, 5)}$.



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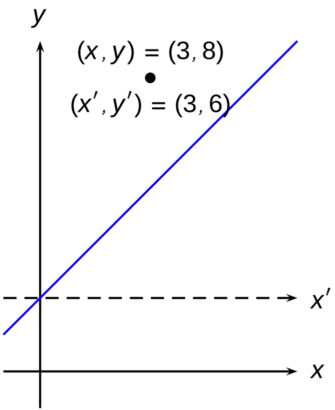

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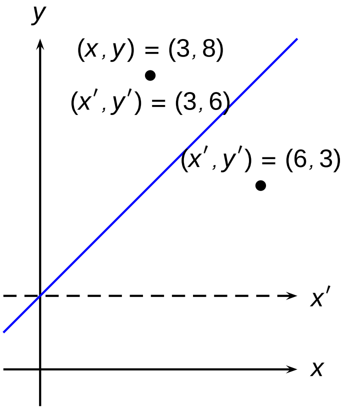

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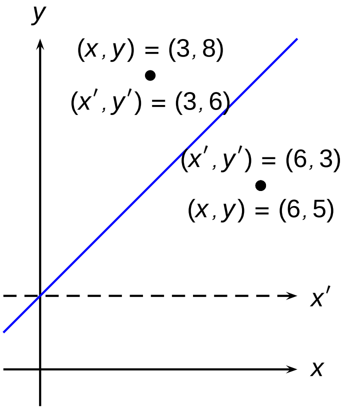


Problem 5

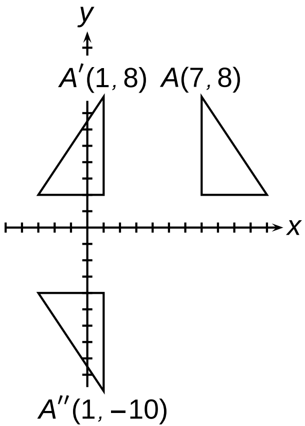
How many degrees clockwise does the hour hand of a 12-hour clock rotate in a 90-minute period?

An hour hand moves 360° in 12 hours. Also, $90\,\mathrm{min} = 1.5\,\mathrm{hr}$. Therefore, the number of degrees moved is

$$\frac{360^\circ}{12\,\mathrm{hr}} \times 1.5\,\mathrm{hr} = 30 \times 1.5 = \boxed{45^\circ}$$

Problem 6

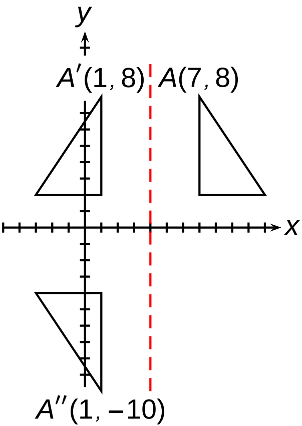
One reflection moves triangle ABC to triangle $A'B'C'$, and then a second reflection moves triangle $A'B'C'$ to triangle $A''B''C''$. A single rotation of triangle ABC moves it to triangle $A''B''C''$. What ordered pair is the center of the rotation?



Problem 6 Solution: Method 1

Since ABC and $A'B'C'$ have identical y coordinates, the first reflection must have been about a line x = constant. The x-coordinate of the line must be the average of the x-coordinates of A and A':

$$x = \frac{1+7}{2} = 4$$



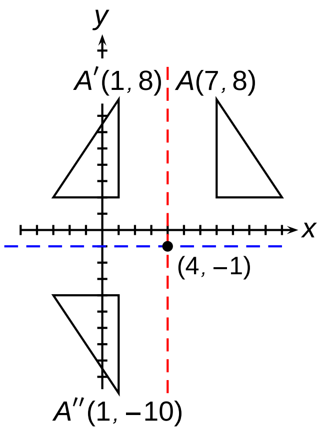
Problem 6 Solution, Method 1 Continued

Since ABC and $A'B'C'$ have identical y coordinates, the first reflection must have been about a line x = constant. The x-coordinate of the line must be the average of the x-coordinates of A and A':

$$x = \frac{1+7}{2} = 4$$

Since $A'B'C'$ and $A''B''C''$ have identical x coordinates, the second reflection must have been about a line y = constant. The y-coordinate of the line must be the average of the y-coordinates of A' and A'':

$$y = \frac{8+(-10)}{2} = -1$$

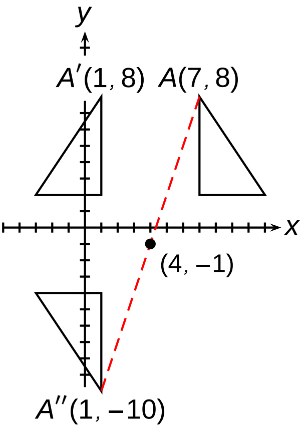


Problem 6 Solution, Method 2

The center of rotation must be the midpoint of the line segment connecting A and A''. The midpoint coordinates will be the average of the coordinates of the two points:

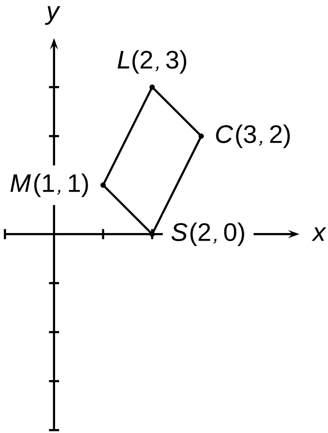
$$(x, y) = \left(\frac{7+1}{2}, \frac{8+(-10)}{2}\right)$$

$$= \boxed{(4, -1)}$$



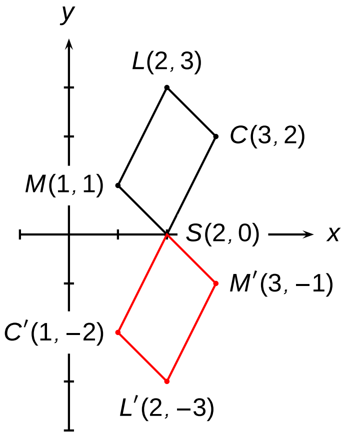
Problem 7

Parallelogram $MSCL$ has coordinates $M(1,1)$, $S(2,0)$, $C(3,2)$ and $L(2,3)$. The parallelogram is rotated 180° about point S to create a second parallelogram $M'S'C'L'$. Segment CM' and segment $C'M$ are then drawn. What is the area of hexagon $MLCM'L'C'$?



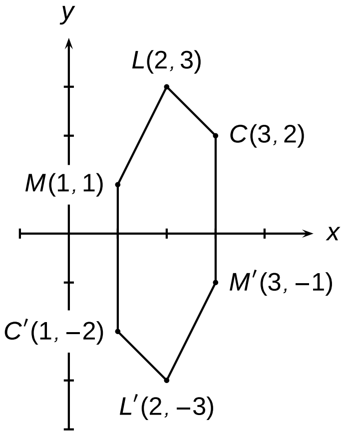
Problem 7, Continued

The y-coordinates of the primed points are just the negatives of the original points. The x-coordinates are found by reflecting through the line $x = 2$, so $x = 1 \to x = 3$ and $x = 3 \to x = 1$.



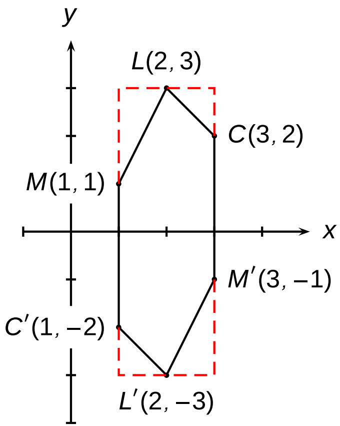
Problem 7, Continued

We now add the segments $C'M$ and CM' and consider how to find the area of the hexagon.



Problem 7, Continued

We can find the area of the enclosing rectangle and then subtract off the area of the added triangles.



Rectangle area $= 2 \times 6 = 12$

Area of 4 triangles $= 1 \times 2 + 1 \times 1 = 3$

Area of hexagon $= 12 - 3 = \boxed{9}$