Workout 7 Solutions

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Problem 1

In 1970, the median age of a man marrying for the first time was 23.2 years. By 1998, the median age rose to 26.7 years. Assuming that the same linear trend continues, what will be the median age for a man marrying for the first time in 2005? Express your answer as a decimal to the nearest tenth.

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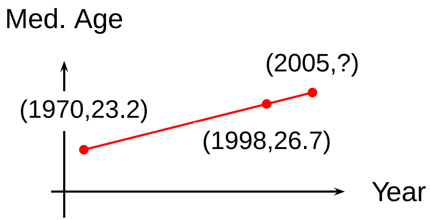

Problem 1, Continued

The median age increased by $26.7 - 23.2 = 3.5$ years over a period of 1998 – 1970 = 28 years. This is a rate of change of $3.5/38 = 1/8$ year of median age per year of elapsed time. If the trend continues, then in 2005 (after 2005 – 1998 – 7 more years), the median age will have increased by $7 \times \frac{1}{8} = 0.875$ years to a new value of $26.7 + 0.875 = 27.575 \approx \boxed{27.6}$ years.

Problem 2

James wants to take summer classes but wants to go to class only between 9 a.m. and noon each day. The table below lists the available courses and class times. What is the maximum number of different courses James can take?

Course	**Days**	**Times (am)**	**Course**	**Days**	**Times (am)**
Archery	MWF	9–9:50	Karate	MWF	9–10:20
Archery	TTh	9–10:20	Karate	TTh	9–10:50
Cartooning	MWF	11–11:50	Drama	MWF	9–9:50
Diving	MTWThF	11–11:50	Physics	TTh	8:30–9:50
Ceramics	MWF	10–10:50	Math Magic	TTh	10:30–11:50

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James can take at most 3 50-minute classes on Monday, Wednesday, and Friday.

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James can take at most 3 50-minute classes on Monday, Wednesday, and Friday. He can take at most 2 80-minute classes on Tuesdays and Thursdays. So the maximum number of classes he can take is 5.

Problem 3

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$$\text{Avg. Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

$$= \frac{2500\,\text{mi} + 3500\,\text{mi}}{5.5\,\text{hr} + 7\,\text{hr}}$$

$$= \frac{6000\,\text{mi}}{12.5\,\text{hr}}$$

$$= \boxed{480\,\text{mi/hr}}$$

Problem 4

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Since five dozen equals 60, then we could obtain the five dozen cookies by buying only Munchies, in groups of a dozen, or by buying only Crunchies, in units of five. So whichever vendor charges the least per cookie should be selected.

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Munchies charges $157/12 \approx 13.08$ cents per cookie. Crunchies charges $60/5 = 12$ cents per cookie. Therefore, we should purchase the cookies from Crunchies at a cost of

$$12\,\text{cents/cookie} \times 60\,\text{cookies} = \boxed{720\,\text{cents}} = \boxed{\$7.20}$$

Problem 5

Corn costs 99 cents per pound, and beans cost 45 cents per pound. If Shea buys 24 total pounds of corn and beans, and it costs \$18.09, how many pounds of corn did Shea buy? Express your answer as a decimal to the nearest tenth.

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Let C and B be the number of pounds of corn and beans, respectively. We are told that

$$C + B = 24$$
$$99C + 45B = 1809$$

We want to obtain an equation that involves only C. Since the second equation contains a term $45B$, then we multiply the first equation by -45 to obtain $-45C - 45B = -1080$. Adding this to the first equation yields

$$(99-45)C+(45-45)B = 1809-1080 \implies 54C = 729 \implies C = \boxed{13.5}$$

Problem 6

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A palindrome is a number (or word) that reads the same backwards as forwards. Since the product ends in 5, one of the numbers must end in 5 and the other must end in an odd digit. Since they are palindromes, the number ending in 5 must also begin with 5. Lets try dividing the product by various numbers of the form $5n5$:

Problem 6, Continued

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$$436{,}995 \div 565 = 773.44\ldots$$

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$$436{,}995 \div 575 = 759.99\ldots$$

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$$436{,}995 \div 585 = 747$$

so the sum of the two palindromes is

$$585 + 747 = \boxed{1332}$$

Problem 7

Take two sheets of $8\frac{1}{2}$ inch by 11 inch paper. Fold one sheet vertically into fourths to form the sides of a rectangular prism. Fold the other sheet horizontally into fourths to form the sides of a different rectangular prism. How much more volume than the smaller prism does the larger prism have? Express your answer as a decimal to the nearest tenth.

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The volume of such a prism or square cylinder is the product of the square base area and the height. So

$$V_1 = 11 \times \left(\frac{8.5}{4}\right)^2 = 11 \times 4.5156 \approx 49.6719$$

$$V_2 = 8.5 \times \left(\frac{11}{4}\right)^2 = 8.5 \times 7.5625 \approx 64.2813.$$

The difference is

$$V_2 - V_1 \approx 64.2813 - 49.6719 \approx \boxed{14.6}$$

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The probability of obtaining a head on any one flip of the coin is $\frac{1}{2}$, the same as obtaining a tail. So the probability of any particular outcome, say "HTHTHTHTT" is equal to

$$\underbrace{\frac{1}{2} \times \frac{1}{2} \times \cdots \times \frac{1}{2}}_{\text{9 factors}} = \left(\frac{1}{2}\right)^9 = \frac{1}{512}$$

A successful outcome will result in 6, 7, 8, or 9 heads. We need to count the number of successful outcomes and multiply this by $1/512$ to calculate the probability of success.

Problem 8, Continued

9 Heads There is only 1 outcome that will result in 9 Heads.

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8 Heads There are 9 ways to obtain 8 heads, because there are 9 positions that could be occupied by the single tail. Another way to look at this is that we are seeking the number of ways to choose the 8 locations of the tails from the 9 possible "slots." This is just the number of combinations of 9 things taken 8 at a time ("9 choose 8") which we write as ${}_9C_8$ or $\binom{9}{8}$, where

$${}_9C_8 = \binom{9}{8} = \frac{9!}{8!\,(9-8)!} = \frac{9!}{8!} = 9.$$

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6 Heads The number of outcomes with 6 heads is ${}_9C_6 = \frac{9!}{6!\,3!} = \frac{9\times 8\times 7}{3\times 2} = 84$.

Problem 8, Continued

So the number N of ways of obtaining 6 or more heads on 9 tosses of the fair coin is

$$
\begin{aligned}
N &= {}_9C_6 + {}_9C_7 + {}_9C_8 + {}_9C_9 \\
&= \frac{9!}{6!\,3!} + \frac{9!}{7!\,2!} + \frac{9!}{8!\,1!} + \frac{9!}{9!\,0!} \\
&= \frac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} + \frac{9 \cdot 8}{2 \cdot 1} + \frac{9!}{9!} \\
&= 84 + 36 + 9 + 1 = 130
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$$= \frac{9!}{6!\,3!} + \frac{9!}{7!\,2!} + \frac{9!}{8!\,1!} + \frac{9!}{9!\,0!}$$

$$= \frac{9\cdot 8\cdot 7}{3\cdot 2\cdot 1} + \frac{9\cdot 8}{2\cdot 1} + \frac{9!}{9!}$$

$$= 84 + 36 + 9 + 1 = 130$$

and the probability of this happening is $N/512$ or

$$\frac{130}{512} = \frac{65}{256} \approx \boxed{0.254}$$

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$$3x + 3y = 3 \implies x + y = 1$$

and if we multiply this last equation by 888 we get
$888x + 888y = 888$.

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and, since $x + y = 1$, then $x = 1 - y = 1 - 2 = -1$ so that

$$x - y = -1 - 2 = \boxed{-3}$$

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There are four general arrangents of boy/girl to consider: BGBGBG, GBGBGB, BGGBGB, and BGBGGB.

For any one of these four basic arrangements, the boys can be rearranged (permuted) among themselves in $3 \times 2 \times 1 = 6$ different ways. Similarly, the three girls can be rearranged among themselves in 6 ways. Since the boys and girls can be rearranged independently, the number of ways to create any one of the four basic arrangements is $6 \times 6 = 36$.

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So the total number of ways that 3 boys and 3 girls can line up so that no two of the three boys are next to each other is

$$4 \times 36 = \boxed{144}.$$