Workout 5 Solutions

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Quiz, December 8, 2004

Problem 1

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If Marika makes 10 more shots in a row, she will have made 20 shots out of 30 attempts. This represents a shooting percentage of $66\frac{2}{3}\%$, more than necessary.

Add'l Made Shots	Total Made	Total Attempts	%
10	20	30	66.7
9	19	29	65.5
8	18	28	64.3
7	17	27	63.0
6	16	26	61.5
5	15	25	60

So Marika needs to make $\boxed{5}$ more shots in a row to hit 60% of her shots.

Problem 2

Five girls are each given two standard dice. Each of the girls rolls her pair of dice and writes down the product of the two numbers that she rolled. What is the probability that all of the girls have written down a product greater than 10? Express your answer as a decimal to the nearest ten thousandth.

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The following table shows the possible outcomes for the product of the two numbers:

×	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

A product greater than 10 is obtained in 17 out of 36 cases. So the probability for a single girl to obtain such a product is

$$\frac{17}{36}$$

Problem 2, Continued

Since each girl has a probability $\frac{17}{36}$ of obtaining a product greater than 10, and since each girl's outcome is independent of the others, the probability that all five of them achieve the desired outcome is the product of the individual probabilities, or

$$\left(\frac{17}{36}\right)^5 \approx \boxed{0.0235}$$

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$$\frac{1}{100\pi} \approx \frac{1}{100 \times 3.14159} = \frac{1}{314.159} \approx \boxed{3.18 \times 10^{-3}} = \boxed{0.00318}$$

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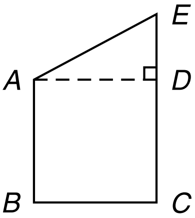
Since the boys and girls can be chosen independently, the fundamental principle of counting says the total number of choices is given by the product:

$$3 \times 10 = \boxed{30}$$

Problem 5

In the diagram, $ABCD$ is a square, $AD = 30\,\text{cm}$, and $AE = 34\,\text{cm}$. What is the area of trapezoid $ABCE$?



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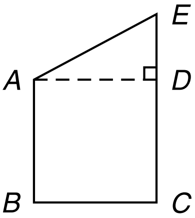

Since $\triangle ADE$ is a right triangle, we can use the Pythagorean theorem to find DE:

$$DE = \sqrt{AE^2 - AD^2} = 34^2 - 30^2 = \sqrt{1156 - 900} = \sqrt{256} = 16$$

The area of $\triangle ADE$ is

$$\frac{1}{2}bh = \frac{1}{2}(DE)(AD) = \frac{1}{2}(16)(30) = (8)(30) = 240$$

Adding this to the area $30^2 = 900$ of square $ABCD$ yields the area of the trapezoid: $900 + 240 = \boxed{1140}$.

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The sum of the seven integers is $7 \times 5 = 35$. If one of the seven takes the greatest possible value, then each of the others must take the least possible value of 1. Let n be the integer with the greatest possible value. Then

$$n = \text{Sum of all 7 integers} - \text{Sum of other six integers}$$

$$= 35 - 6 \times 1 = 35 - 6 = \boxed{29}$$

Problem 7

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The first such integer is 100. The last is 999. There are $999 - 100 + 1 = 900$ such integers. The sum is an arithmetic series, equal to the average of the first and last term, multiplied by the number of terms:

$$\begin{aligned} 100 + 101 + 102 + \cdots + 999 &= \frac{100 + 999}{2} \times 900 \\ &= 1099 \times 450 \\ &= \boxed{494{,}550} \end{aligned}$$

Problem 8

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If every player played every other player once, the number of matches would be

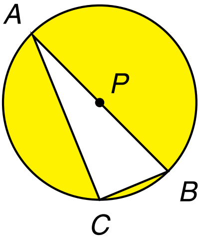
$$\binom{10}{2} = \frac{10!}{2!\,(10-2)!} = \frac{10!}{2!\,8!} = \frac{10 \times 9}{2} = 5 \times 9 = 45$$

Since 900 matches were actually played, then

$$N = \frac{900}{45} = \boxed{20}$$

Problem 9

Segment AB measures 4 cm and is a diameter of circle P. In $\triangle ABC$, point C is on circle P and $BC = 2$ cm. What is the area of the yellow region? Express your answer as a decimal to the nearest tenth.



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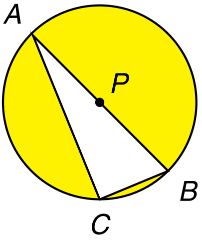

We use the fact that any triangle inscribed in a circle, one side of which is a diameter of the circle, must be a right triangle. Then

$$AC = \sqrt{AB^2 - BC^2} = \sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}$$

and the area of the triangle is

$$A_\triangle = \frac{1}{2}bh = \frac{1}{2}(2)(2\sqrt{3}) = 2\sqrt{3}$$

so the shaded area is

$$\pi r^2 - A_\triangle = 4\pi - 2\sqrt{3} \approx \boxed{9.1}$$

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Method 1

$$(3x - 4) - (5 - 6x) = 3x + 6x - 4 - 5 = 9x - 9$$

which takes its maximum value at $x = 8$:

$$9(8) - 9 = 72 - 9 = \boxed{63}$$

Method 2

Since each of the expressions is a straight line, the maximum difference will occur at one of the endpoints:

x	$3x - 4$	$5 - 6x$	Difference
5	11	-25	36
8	20	-43	$\boxed{63}$