

Workout 4 Solutions

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Quiz, November 10, 2004

Problem 1

The graph below shows the top five vote-recipients from a poll of 5000 people at Westfield Middle School. To the nearest 10, how many people either voted for Michael Jordan or Tiger Woods to endorse a product?

Michael Jordan	Tiger Woods	Lance Armstrong	Shaquille O'Neal	Brett Favre
23.9	13.7	4.1	2.9	1.8

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The percentage of people voting for either Jordan or Woods is

$$23.9\% + 13.7\% = 37.6\%$$

The number voting for either of these two is found by multiplying the percentage times the total number of voters:

$$37.6\% \times 5000 = 37.6 \times 50 = \boxed{1880}$$

and no rounding is needed.

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Since $A \neq B$, then either $A > B$ or $A < B$. Let's assume for definiteness that $A > B$. Then “ AB ” $>$ “ BA ” and we form the difference “ AB ” $-$ “ BA ”. Note that the ones digit of the difference will be $10 + B - A$, because a carry is required. The tens digit will be $A - 1 - B$. Multiplying the tens digit by 10 and adding it to the ones digit yields the difference:

$$\begin{aligned}\text{“}AB\text{”} - \text{“}BA\text{”} &= 10(A - 1 - B) + (10 + B - A) \\ &= 10A - 10 - 10B + 10 + B - A \\ &= 9A - 9B = 9(A - B)\end{aligned}$$

so that 9 will be a factor of the difference. Since 3 is a prime factor of 9, it will be a factor of the difference, too.

Problem 3

The winner of the local triathlon swam 0.5 miles, biked 15 miles and ran 3.5 miles. He finished the race in 1 hour, 2 minutes and 2 seconds. For the entire triathlon, what was his average speed in feet per second? Express your answer to the nearest whole number.

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$$\begin{aligned}\text{Avg Speed} &= \frac{\text{Distance}}{\text{Time}} = \frac{19 \text{ mi}}{62 \text{ min} + 2 \text{ sec}} \\ &= \frac{19 \text{ mi}}{62 \text{ min} \times 60 \text{ sec/min} + 2 \text{ sec}} \\ &= \frac{19 \text{ mi}}{3722 \text{ sec}} \times 5280 \text{ ft/mi} \\ &\approx 26.953 \text{ ft/sec} \approx \boxed{27 \text{ ft/sec}}\end{aligned}$$

Problem 4

Mady's Messenger Service had total sales of \$10,000 this week. Studies have shown that if Mady's does not advertise during the week, the week's sales total will be down 5% from the previous week's sales total. If Mady's does not advertise for the next four weeks, what are the expected sales for the fourth week?

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Each week of no advertising will result in sales of 95% of the previous week. So at week 4, the sales will be

$$\begin{aligned}0.95 \times 0.95 \times 0.95 \times 0.95 \times 10,000 &= (0.95)^4 \times 10,000 \\ &= 8145.0625 \\ &\approx \boxed{\$8145.06}\end{aligned}$$

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Method 1

$$\begin{aligned}f\left(\frac{3}{4}\right) &= 8\left(\frac{3}{4}\right)^3 - 6\left(\frac{3}{4}\right)^2 + \frac{12 \times 3}{4} - 9 \\&= \frac{8 \times 27}{64} - \frac{6 \times 9}{16} + 9 - 9 \\&= \frac{27}{8} - \frac{27}{8} = \boxed{0}\end{aligned}$$

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Method 2

Write $f(x) = [(8x - 6)x + 12]x - 9$ (Horner's Rule). Then

$$\begin{aligned}f\left(\frac{3}{4}\right) &= \left[\left(\frac{8 \times 3}{4} - 6\right) \frac{3}{4} + 12\right] \frac{3}{4} - 9 \\&= [12] \frac{3}{4} - 9 = 9 - 9 = \boxed{0}\end{aligned}$$

Problem 6

“Buy 3, Get 2 Free!” is equivalent to purchasing the five items at a discount of what percent?

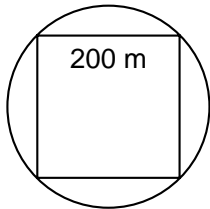
Problem 6

“Buy 3, Get 2 Free!” is equivalent to purchasing the five items at a discount of what percent?

The cost is $\frac{3}{5}$ or 60% of the normal price. So this represents a discount of 40%.

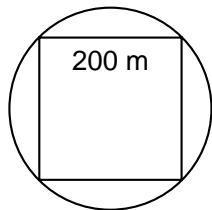
Problem 7

A circular field circumscribes a plot of land that is 200 meters square, as shown. How many square meters of the circular region are not inside the square region? Express your answer to the nearest whole number.



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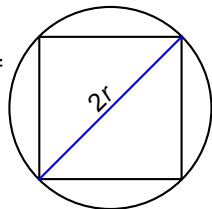


By the Pythagorean theorem,

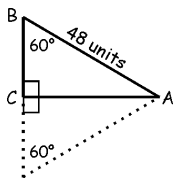
$$2r = 200\sqrt{2} \implies r = 100\sqrt{2}$$

The desired area is the difference between that of the circle and that of the square:

$$\begin{aligned}\Delta A &= \pi(100\sqrt{2})^2 - 200^2 = 20000\pi - 40000 \\ &= 20000(\pi - 2) \approx 20000(3.1416 - 2) \\ &= 20000 \times 1.1416 \approx \boxed{22,832}\end{aligned}$$

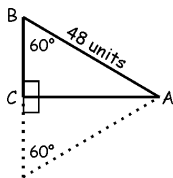


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ABC is one-half of an equilateral triangle, so

$$BC = \frac{1}{2}AB = \frac{48}{2} = 24.$$

We can use the Pythagorean theorem to find AC :

$$AC = \sqrt{AB^2 - BC^2} = \sqrt{48^2 - 24^2} = 24\sqrt{3}$$

So the perimeter is

$$48 + 24 + 24\sqrt{3} \approx 113.57 \approx \boxed{114}$$

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Distance traveled is proportional to speed, and also to time. So

$$\begin{aligned}\text{Old Distance} &= \text{Old Speed} \times \text{Old Time} \\ \text{New Distance} &= \text{New Speed} \times \text{New Time} \\ \frac{\text{New Distance}}{\text{Old Distance}} &= \frac{\text{New Speed} \times \text{New Time}}{\text{Old Speed} \times \text{Old Time}} \\ &= \frac{\text{New Speed}}{\text{Old Speed}} \times \frac{\text{New Time}}{\text{Old Time}} \\ &= 2 \times \frac{2}{3} = \frac{4}{3}\end{aligned}$$

So the new distance is $\frac{4}{3} \times 300 = \boxed{400 \text{ mi}}$

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$$\text{Cyl Volume} = \text{Area of Base} \times \text{Height}$$

$$= \pi r^2 h$$

$$= \pi \times 2.5^2 \times 5 = 31.25\pi \approx \boxed{98}$$