Workout 3 Solutions

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Problem 1

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Any number that is divisible by both 11 and 5 must be divisible by $11 \times 5 = 55$. Three-digit multiples of 55 are

$$110, 165, 220, \ldots, 990$$

. There are

$$\frac{990 - 110}{55} + 1 = \boxed{17}$$

such numbers.

Problem 2

Twenty-five teachers were asked the length, in minutes, of their daily drive to school. Their responses are recorded in this stem-and-leaf plot. What percent of the teachers take 25 or more minutes to get to school?

Driving Time to School (minutes)

0	5	6	6	7	9	
1	0	1	2	8	9	9
2	0	2	3	4	5	7
3	1	2	2	2	2	8
4	1	3				

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There are 10 out of the 25 teachers taking 25 or more minutes:

$$\frac{10}{25} = \frac{40}{100} = \boxed{40\%}$$

Problem 3

The integer 49 can be written as the sum of smaller perfect squares in a variety of ways. One such way includes six terms: $25 + 9 + 9 + 4 + 1 + 1$. If each term has a value between 0 and 49, what is the fewest number of perfect-square terms that can be added together for a sum of 49?

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The perfect square numbers less than 49 are 36, 25, 16, 9, 4, and 1. To for a sum of the fewest terms, let's try starting with the largest number in this list: 36. We now must express 49 – 36 = 13 as a sum of perfect squares: 13 = 9 + 4. So

$$49 = 36 + 9 + 4$$

This turns out to be the fewest number of terms possible: $\boxed{3}$.

Problem 4

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$$\text{Area} = 18{,}613{,}087\,\text{people} \times \frac{1\,\text{sq. mi}}{6.27\,\text{people}}$$

$$\approx 2{,}968{,}594\,\text{sq. mi} \approx \boxed{2{,}970{,}000\,\text{sq. mi}}$$

Problem 5

Two cars leave Arlington on I-20, one traveling due east and one traveling due west. One car is moving at a rate of 37 miles per hour, and the other is traveling at a rate of 41 miles per hour. How many miles from each other will they be in four hours?

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The relative speed with which the cars are receding from each other is $37 + 41 = 78$ miles per hour.

$$\begin{aligned} \text{Distance} &= \text{Speed} \times \text{Time} \\ &= 78\,\text{mi/hr} \times 4\,\text{hr} \\ &= \boxed{312\,\text{mi}} \end{aligned}$$

Problem 6

The mean radius of Mars is 2106 miles and the mean radius of Pluto is 707 miles. The volume of a sphere is $\frac{4}{3}\pi r^3$ with radius r units. Assuming that the planets are perfect spheres, what is the volume of Mars divided by the volume of Pluto? Express your answer to the nearest whole number.

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$$\frac{V_{\text{Mars}}}{V_{\text{Pluto}}} = \frac{\frac{4}{3}\pi r_{\text{Mars}}^3}{\frac{4}{3}\pi r_{\text{Pluto}}^3} = \frac{r_{\text{Mars}}^3}{r_{\text{Pluto}}^3} = \left(\frac{2106}{707}\right)^3 \approx 2.98^3 \approx \boxed{26}$$

Problem 7

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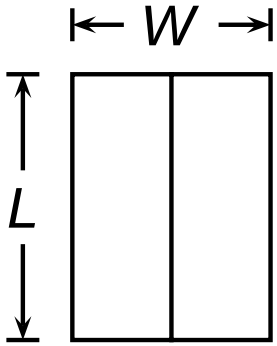
If the machine keeps 8.9 cents per dollar, then it returns $100 - 8.9 = 91.1$ cents per dollar inserted. Thus, Ashley's return is

$$\frac{91.1}{100} \times 50.5 = 0.911 \times 50.5 = 46.0055 \approx \boxed{\$46.01}$$

Problem 8

Keli and Mario are planning to plant rectangular gardens of the same length, side by side with fencing all around and dividing the two plots. The total amount of fencing is 100 feet. If the total area of the two plots is 336 square feet and the dimensions are integers, what is the length of the fence that divides the two plots?



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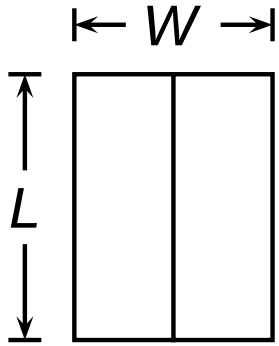

The total length of fencing used is $3L + 2W = 100$. The area condition tells us that $LW = 336$ or $W = 336/L$. Substituting this value of W into the first equation yields

$$100 = 3L + 2\frac{336}{L} = 3L + \frac{672}{L}$$

We can now see that L must be less than 33 and greater than 7 (why?). A few guess and check tries yields $L = \boxed{24}$.

Problem 9

At sea level, air pressure is 14.7 pounds per square inch. If the pressure under water increases by $4.3\,\text{lb/in}^2$ for every 10 feet you descend, how many times greater is the pressure at 400 feet below sea level than at sea level? Express your answer as a decimal to the nearest tenth.

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The pressure at a depth of 400 feet is

$$14.7 + 4.3 \times \frac{400}{10} = 14.7 + 4.3 \times 40 = 14.7 + 172 = 186.7$$

pounds per square inch. The ratio of this pressue to atmospheric pressure is

$$\frac{186.7}{14.7} \approx \boxed{12.7}$$

Problem 10

A circular disk has a radius of two units. A point is marked on the edge of the disk. The disk rotates about its center, causing the point to travel a distance of 90 units. How many rotations did the disk make? Express your answer as a decimal to the nearest tenth.

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The circumference of the disk is $C = 2\pi r = 4\pi$. The number of rotations is therefore

$$\frac{90}{C} = \frac{90}{4\pi} \approx 7.162 \approx \boxed{7.2}$$