

# Workout 1 Solutions

Peter S. Simon

October 13, 2004

## Problem 1

Five friends are standing in a line. Cliff is standing directly behind Danny, and there are two people between Cliff and Mark. Mark is standing somewhere behind Eric, but somewhere in front of Tom. Which of the five friends is fourth in line?

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We know that Cliff is directly behind Danny, and there are two places between Cliff and Mark. So the possibilities are:

(1) 

Danny	Cliff			Mark
-------	-------	--	--	------

(2) 

Mark		Danny	Cliff	
------	--	-------	-------	--

(3) 

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Since Mark is behind Eric but in front of Tom, only (3) can be right. The solution is

Eric	Mark	Tom	Danny	Cliff
------	------	-----	-------	-------

putting 

Danny
-------

 in fourth place.

## Problem 2

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Joe and Frank scored  $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$  of the points. So Ken and Mike's total of 4 points constitute  $\frac{1}{6}$  of the total team points. Therefore, the team scored  $6 \times 4 = \boxed{24}$  points.

## Problem 3

Lotta Dough used a \$100 bill to pay for two books that cost \$11.98 each, two videos that cost \$14.49 each and miscellaneous school supplies that cost \$24.17. If the tax rate is 7.25%, how much does Lotta receive in change from her \$100?

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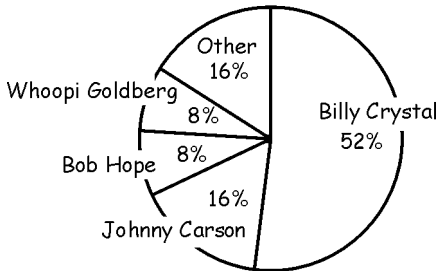
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$$\text{Change} = \$100 - \$82.70 = \boxed{\$17.30}$$

## Problem 4

The circle graph shows the results of a 2002 survey of 4050 people. How many people chose Billy Crystal as the best Oscars host? Express your answer to the nearest hundred.

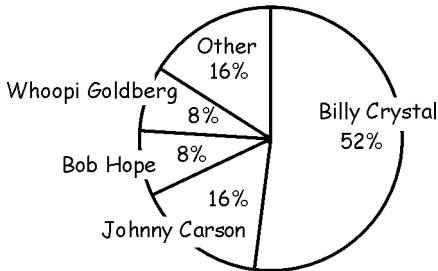
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$$\# \text{ People} = \frac{52}{100} \times 4050 = \boxed{2106}$$

## Problem 5

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In 2004, the father's age is 32. Let  $x$  be the # years needed.

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$$32 + x = 3(8 + x) = 24 + 3x \implies 2x = 32 - 24 = 8 \implies x = 4$$

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### Method 2

Year	Marika's Age	Father's Age
2004	8	32
2005	9	33
2006	10	34
2007	11	35
2008	12	36

## Problem 6

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$$\text{Distance} = \frac{15 \text{ mi}}{1 \text{ hr}} \times \left( 4 \text{ min} \times \frac{1 \text{ hr}}{60 \text{ min}} \right) = \boxed{1 \text{ mi}}$$

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$$\begin{aligned} \# \text{ Cans} &= (\text{Avg. of 1st and last terms}) \times (\# \text{ of terms}) \\ &= \frac{1 + 28}{2} \times 10 = 29 \times 5 = \boxed{145} \end{aligned}$$

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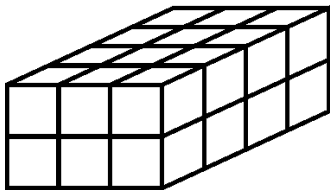
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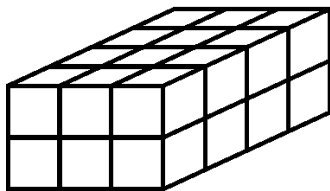
## Problem 9

A 2 by 3 by 4 rectangular prism is painted and then cut into 24 unit cubes. If a unit cube will be selected at random, what is the probability that it will have fewer than two painted faces? Express your answer as a common fraction.



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All blocks have at least one face painted. The only blocks with 1 painted face are the two center blocks on the top layer and the two center blocks on the bottom layer for a total of 4 blocks. So the probability of selecting one of these four at random is

$$P = \frac{4}{24} = \boxed{\frac{1}{6}}$$

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The cost of six acres will be one-third the cost of eighteen acres or

$$\text{Cost} = \frac{\$27,766.80}{3} = \$9,255.60 \approx \boxed{\$9,256}$$