Warm-Up 8 Solutions

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Problem 1

Rodney uses the following clues to try to guess a secret number:

It is a two-digit integer.

The tens digit is odd.

The units digit is even.

The number is greater than 65.

If Rodney guesses a number that has each of these properties, what is the probability that Rodney will guess the correct number? Express your answer as a common fraction.

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If Rodney guesses a number that has each of these properties, what is the probability that Rodney will guess the correct number? Express your answer as a common fraction.

The tens digit can be either 7 or 9. The ones digit can be 0, 2, 4, 6, or 8. With 2 choices for the tens digit and 5 choices for the ones digit, the number of possibilities is $2 \times 5 = 10$. The probability that Rodney has guessed the right number is therefore $\boxed{\frac{1}{10}}$

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$$\begin{aligned}
\text{Avg. Score} &= \frac{\text{Total Points Scored}}{\text{\# Games Played}} \\
&= \frac{17 \times 8 + 24 \times 6}{14} \\
&= \frac{280}{14} = \boxed{20}
\end{aligned}$$

Problem 3

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$$\begin{aligned}(4 \# 5) \# 6 &= (2 \times 4 - 3 \times 5) \# 6 \\ &= (8 - 15) \# 6 \\ &= -7 \# 6 = 2 \times (-7) - 3 \times 6 \\ &= -14 - 18 \\ &= \boxed{-32}\end{aligned}$$

Problem 4

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$$x + 5 < 8 \implies x < 8 - 5 = 3.$$

Since the only prime number less than 3 is $\boxed{2}$, this is the value of x.

Problem 5

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The exterior angles of any polygon must add up to 360°, since the polygon is closed. Each exterior angle E is related to its interior angle I via $E = 180^\circ - I$. So, for a heptagon, we have

$$360^\circ = 7 \times 180^\circ - \text{Sum of Interior Angles}$$

$$\boxed{\text{Sum of Interior Angles} = 7 \times 180^\circ - 360^\circ = (7-2) * 180^\circ}$$

$$7x + 4^\circ = 900^\circ$$

$$7x = 896^\circ$$

$$x = 128^\circ$$

The largest interior angle is $x + 4^\circ = \boxed{132^\circ}$

Problem 6

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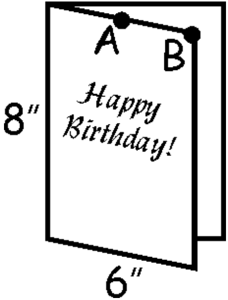
When making change, Steve can include 0 or 1 quarters (2 choices); 0, 1, or 2 nickels (3 choices); and 0, 1, 2, or 3 pennies (4 choices). By the Fundamental Principle of Counting there are

$$2 \times 3 \times 4 = 24$$

different combinations of coins Steve could present as change. Because of the denominations of the coins, no two distinct combinations will result in the same change value, so this is also the number of different amounts of change Steve could present. However, we have to reduce the count by 1, since we included the case of 0 quarters, 0 nickels, and 0 pennies, which is not allowed. So the answer is $\boxed{23}$.

Problem 7

A greeting card is six inches wide and eight inches tall. Point A is three inches from the fold, as shown. As the card is opened to an angle of 45°, through how many more inches than point A does point B travel? Express your answer as a common fraction in terms of π.



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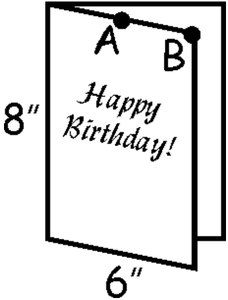

Each point travels one-eighth of the circumference of a circle. For Point A the radius is 3 inches and for Point B it is 6 inches. The difference in distance traveled is

$$\frac{1}{8} \times 2\pi \times 6 - \frac{1}{8} \times 2\pi \times 3 = \frac{\pi}{4} \times (6 - 3) = \boxed{\frac{3\pi}{4}}$$

Problem 8

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$$72 = 2^3 \times 3^2$$

so that 72 has $(3 + 1)(2 + 1) = 12$ factors. If we multiply 72 by any prime number p other than 2 or 3, the number of factors will be $12 \times 2 = 24$ which is too great. If we multiply it by 2 the number of factors will be $(4 + 1)(2 + 1) = 15$ which doesn't work. However if we multiply it by 3 we find that

$$3 \times 72 = 216 = 2^3 \times 3^3$$

which has $(3 + 1)(3 + 1) = 16$ factors, as desired. The answer is therefore $\boxed{216}$.

Problem 9

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First, let's calculate the number of two-person combinations the teacher could have picked. There are five choices for the first person, and once this choice is made, there are 4 remaining choices for the second person, so there are $5 \times 4 = 20$ ways the teacher could choose two students. However, we have to divide by 2, because we've counted, e.g., (John,Kevin) and (Kevin,John) as different pairs. So there are 10 combinations of 5 things taken 2 at a time:

$$ {}_5C_2 = \frac{5!}{2!(5-2!)} = \frac{5 \times 4}{2} = 10 $$

There is only one combination containing both girls, so the probability of choosing both of them is $\boxed{1/10}$.

Problem 10

Rosalee plans to open a savings account and a checking account. She has decided to deposit a total of $45 per week, such that $20 goes into the checking account each week and the remaining money goes into the savings account. When she has deposited a total of $200 into her savings account, how much will she have deposited into her checking account?

Problem 10

Rosalee plans to open a savings account and a checking account. She has decided to deposit a total of \$45 per week, such that \$20 goes into the checking account each week and the remaining money goes into the savings account. When she has deposited a total of \$200 into her savings account, how much will she have deposited into her checking account?

The ratio of money in the checking account to money in the savings account is

$$\frac{\text{Checking}}{\text{Savings}} = \frac{20}{25} = \frac{4}{5}$$

So the money in the checking account is

$$\frac{4}{5} \times \$200 = \boxed{\$160}$$