Warm-Up 6 Solutions

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Quiz: October 27, 2004

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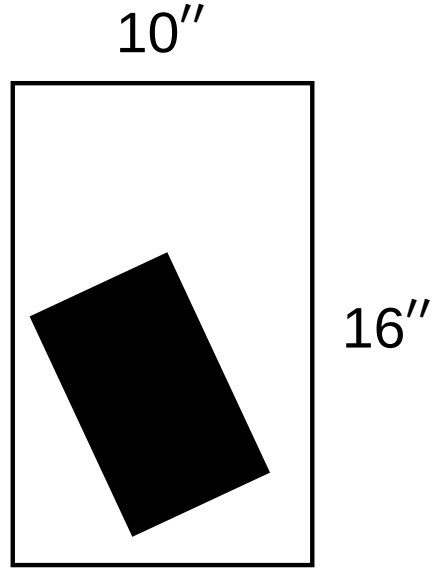
The probability of obtaining these two independent outcomes is the product of the two probabilities:

$$\frac{1}{3} \times \frac{1}{2} = \boxed{\frac{1}{6}}$$

Problem 2

The shaded rectangular region covers one-fourth of the area of the large 10-inch by 16-inch rectangle. What is the number of square inches in the area of the shaded region?



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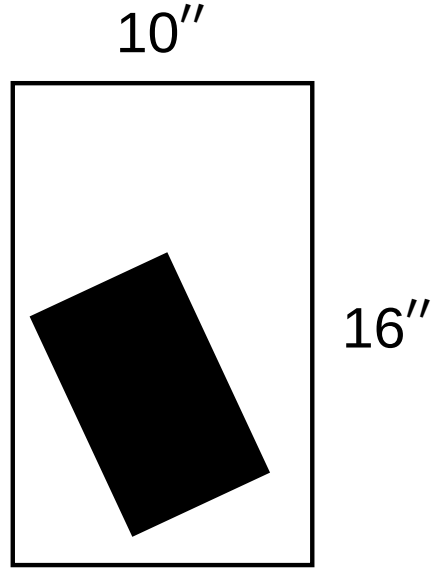

The area of the large rectangle is $10 \times 16 = 160$. The area of the shaded rectangle of one-fourth of this or $\boxed{40}$ square inches.

Problem 3

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Let T and M be the number of coins initially held by Tim and Mike, respectively. Then

$$T = 3M, \quad M + 1 = \frac{T}{2}$$

Eliminating T from the second equation using the first equation yields

$$M + 1 = 3M/2 \implies 2M + 2 = 3M \implies M = 2$$

and since $T = 3M$ then $T = 6$ and $M + T = \boxed{8}$.

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The circumference C and radius r are related by

$$C = 3 = 2\pi r$$

so that

$$r = \frac{C}{2\pi} = \frac{3}{2\pi} = \frac{A}{B\pi}$$

so that $A = 3$ and $B = 2$, and

$$A + B = \boxed{5}$$

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Another way so find T_5

$$\begin{array}{rcccccccc} T_5 & = & 1 & + & 2 & + & \cdots & + & 5 \\ T_5 & = & 5 & + & 4 & + & \cdots & + & 1 \\ \hline 2T_5 & = & 6 & + & 6 & + & \cdots & + & 6 \end{array}$$

Since there are 5 terms in the series, we have that $2T_5 = 5 \times 6$ or $T_5 = \frac{5\times 6}{2} = 5 \times 3 = 15$.

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so there are $\boxed{25}$ digits in this number.

Problem 7

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$$\begin{aligned}\text{Surface Area} &= \frac{1}{2}(4\pi r^2) + \text{Area of Circle} \\ &= 2\pi r^2 + \pi r^2 = 3\pi r^2 \\ &= 3\pi(6^2) = 3 \times 36\pi = \boxed{108\pi}\end{aligned}$$

Problem 8

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$$F = 0.9(2C) + 32 = 1.8C + 32$$

$$= 1.8(20) + 32 = 36 + 32 = \boxed{68^\circ \text{ F}}$$

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The difference must be 4 since $6 - 2 = 10 - 6 = 4$. So

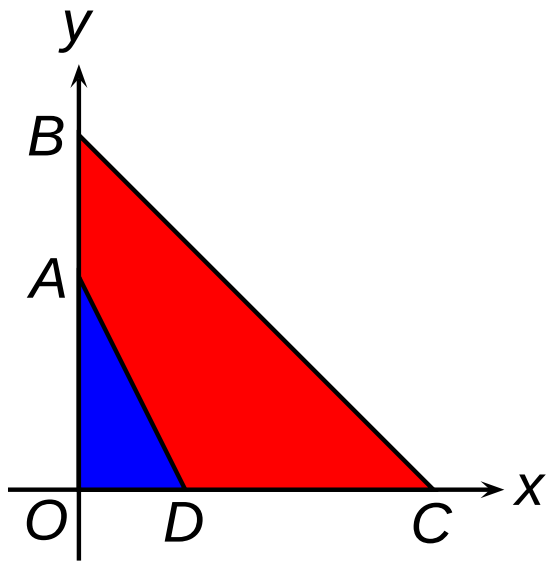
$$y = 26 - 4 = 22, \quad x = y - 4 = 22 - 4 = 18$$

and

$$x + y = 18 + 22 = \boxed{40}$$

Problem 10

The lines $2x + y = 6$ and $x + y = 10$ together with the x-axis and y-axis are drawn on a coordinate grid to form quadrilateral $ABCD$. What is the area of quadrilateral $ABCD$?



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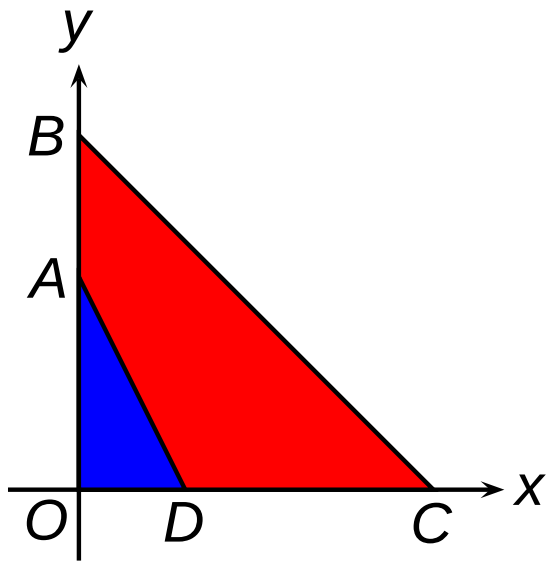

The points are located at

$$A(0, 6), \quad B(0, 10), \quad C(10, 0), \quad D(3, 0)$$

The quadrilateral area can be computed as the difference of triangle OCB's area minus triangle ODA's area:

$$\text{Area} = \frac{1}{2} \times 10 \times 10 - \frac{1}{2} \times 3 \times 6$$

$$= 50 - 9 = \boxed{41}$$