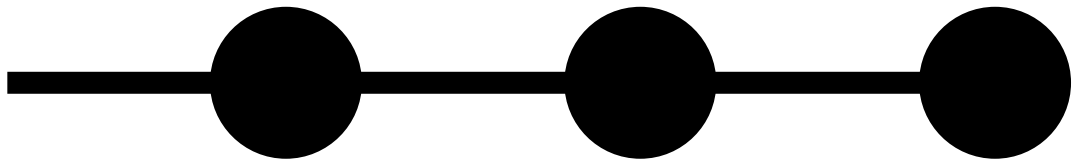
Warm-Up 5 Solutions

Peter S. Simon

October 27, 2004

Problem 1

A set of magnetic strips and balls are connected in an alternating fashion, as shown. The length of the first four items in the chain is 14 cm, and the length of the first seven items is 25 cm. What is the length of the first 25 items in the chain?



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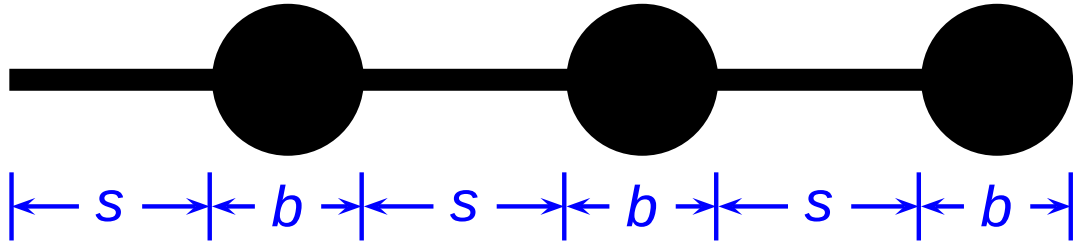

We are given that

$$2(s + b) = 14, \quad 3(s + b) + s = 25$$

If we divide the first equation by 2 we find that $s + b = 7$. The second equation can then be solved for s:

$$s + 3(s + b) = 25 \implies s + 3(7) = 25 \implies s + 21 = 25 \implies s = 4.$$

The length of the first 25 items is then

$$12(s + b) + s = 12(7) + 4 = \boxed{88}$$

Problem 2

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Let x be the original price of a shirt. Then

$$0.65x = \$26 \implies x = \frac{\$26}{0.65} = \boxed{\$40}$$

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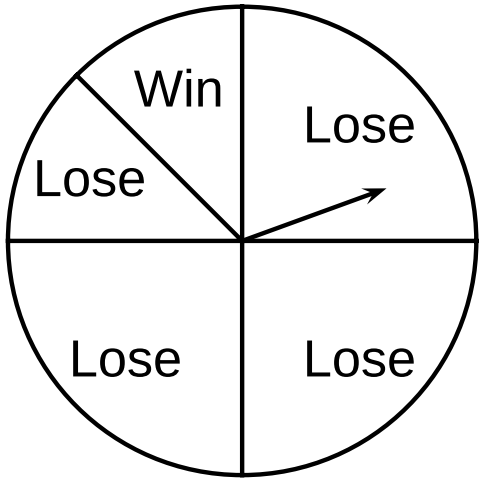
Mar	31	91
Apr	30	121

So day 122 will occur on $\boxed{\text{May 1}}$.

Problem 4

Michael spins the spinner twice. All three of the larger sectors are equal in area and have central angles of 90°. The two smaller sectors have equal area. What is the probability that he will "WIN" on both spins? Express your answer as a common fraction.



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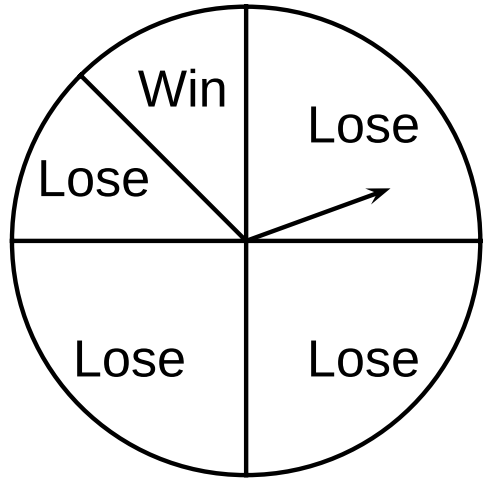

The Win wedge has a central angle of 45°. So the probability of getting a win a one spin is

$$P = \frac{45}{360} = \frac{1}{8}$$

The probability of obtaining two consecutive wins is

$$P \times P = \frac{1}{8} \times \frac{1}{8} = \boxed{\frac{1}{64}}$$

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$$\frac{6\,\text{mi}}{1\,\text{hr}} \times \frac{5280\,\text{ft}}{1\,\text{mi}} \times \frac{1\,\text{hr}}{60\,\text{min}} \times \frac{1\,\text{min}}{60\,\text{sec}} = \boxed{8.8\,\text{ft/sec}}$$

Problem 6

Zach has three bags and a bunch of pencils to be placed into the bags. He is told to place the greatest number of pencils possible into each of the three bags while also keeping the number of pencils in each bag the same. What is the greatest number of pencils he could have left over?

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The question is asking for the largest remainder that can be obtained when dividing an integer by 3, which is, of course, $\boxed{2}$.

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Marta's 24'th birthday will occur $24 - 7 = 17$ years after she turns 7. So Kevin will be $3 + 17 = 20$ when Marta is 24. So Marta's age expressed as a percentage of Kevin's age at that time will be

$$\frac{24}{20} \times 100\% = \boxed{120\%}$$

Problem 8

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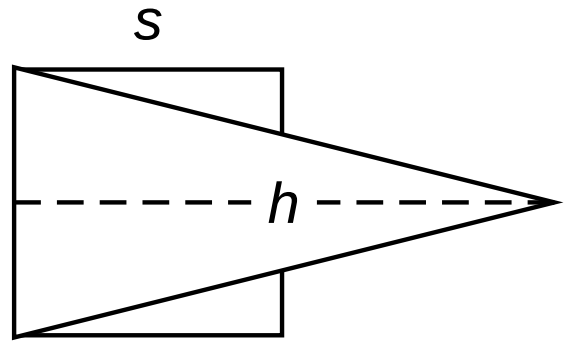
The two-digit primes having a units digit of 7 are:

$$17, 37, 47, 67, 97$$

so there are $\boxed{6}$ such numbers.

Problem 9

The isosceles triangle and the square shown here have the same area in square units. What is the height of the triangle, h, in terms of the side length of the square, s?



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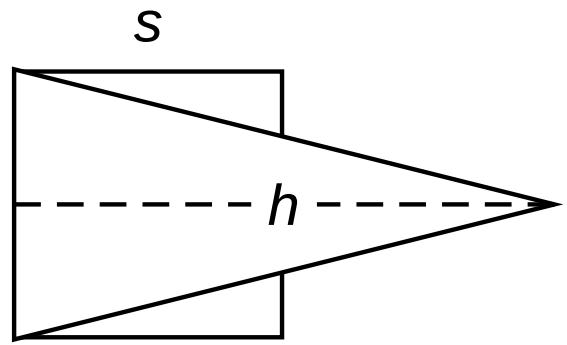

The area of the square is s^2. The area of the triangle is $\frac{1}{2}sh$. If these are equal then

$$\frac{1}{2}sh = s^2 \implies \frac{1}{2}h = s \implies h = \boxed{2s}$$

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There is only 1 such line. This is one of Euclid's postulates.