Warm-Up 3 Solutions

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Problem 1

An automobile insurance company has compiled data from a survey of 1000 16-year-old drivers during the year 2003. According to the results shown, what percent of them have had at least two accidents? Express your answer to the nearest tenth.

# Accidents	# Drivers
0	124
1	234
2	346
3	176
4	120

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2	346
3	176
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$$\frac{\text{\# drivers with} \geq 2 \text{ accidents}}{\text{Total \# drivers}} \times 100\%$$

$$= \frac{346 + 176 + 120}{124 + 234 + 346 + 176 + 120} \times 100\%$$

$$= \frac{642}{1000} \times 100\%$$

$$= \boxed{64.2\%}$$

Problem 2

Connie spent the weekend making cookies. She made 60 sugar cookies, 80 chocolate chip cookies and 100 peanut butter cookies. She plans to make packages of cookies that each contain an identical assortment of whole cookies. How many cookies are in a package, assuming that she makes as many packages as possible and uses all of the cookies she made?

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To maximize the number of packages, we minimize the number of cookies per package. The numbers of each kind of cookie in a package must be in the ratio

$$60 : 80 : 100 = 6 : 8 : 10 = 3 : 4 : 5$$

so that there are $3 + 4 + 5 = \boxed{12}$ cookies per package.

Problem 3

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What is the greatest surface area of a rectangular solid that can be built from 16 identical unit cubes? To maximize the surface area,

we choose an arrangement that maximizes the number of exposed faces on each cube. This will happen when the cubes are arranged in a straight line. In this case, all but two of the cubes will have four faces exposed. The two end cubes will have five faces exposed. So the surface are is

$$14 \times 4 + 2 \times 5 = \boxed{66}$$

Problem 4

On a number line, the coordinates of P and Q are 8 and 48, respectively. The midpoint of $\overline{PQ}$ is B, the midpoint of $\overline{BQ}$ is C, and the midpoint of $\overline{PC}$ is D. What is the coordinate of D?

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$$B \text{ is midpoint of } \overline{PQ} \implies B = \frac{8+48}{2} = 28$$

$$C \text{ is midpoint of } \overline{BQ} \implies C = \frac{28+48}{2} = 38$$

$$D \text{ is midpoint of } \overline{PC} \implies D = \frac{8+38}{2} = \boxed{23}$$

Problem 5

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$$m = \frac{n^2 + 3}{4} = \frac{11^2 + 3}{4} = \frac{124}{4} = \boxed{31}$$

Problem 6

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Method 1: Without Calculator

$$10\pi \approx 10 \times 3.141 \approx 31.4, \quad \sqrt{99} \approx \sqrt{100} = 10$$

$$10\pi + \sqrt{99} \approx 31.4 + 10 = 41.4 \approx \boxed{41}$$

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$$10\pi \approx 10 \times 3.141 \approx 31.4, \quad \sqrt{99} \approx \sqrt{100} = 10$$
$$10\pi + \sqrt{99} \approx 31.4 + 10 = 41.4 \approx \boxed{41}$$

Method 2: With Calculator

$$10\pi + \sqrt{99} \approx 41.3658 \approx \boxed{41}$$

Problem 7

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$$\text{Mean} = \frac{73 + 78 + 81 + 90 + 85 + 97}{6} = \frac{504}{6} = \boxed{84}$$

Problem 8

Christine jogged for half an hour. Amy walked for 50 minutes. Using the information in this chart about exercise and calories burned, how many more calories than Amy did Christine burn?

Activity	Calories Burned in 5 Min.
Walking	28
Jogging	57

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Activity	Calories Burned in 5 Min.
Walking	28
Jogging	57

$$\text{Christine cals} = \frac{57\,\text{cal}}{5\,\text{min}} \times 30\,\text{min} = 342\,\text{cal}$$

$$\text{Amy cals} = \frac{28\,\text{cal}}{5\,\text{min}} \times 50\,\text{min} = 280\,\text{cal}$$

$$\text{Christine cals} - \text{Amy cals} = 342 - 280 = \boxed{62\,\text{cal}}$$

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current ages of Tyler and Mary.

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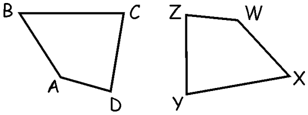
Since we want to find Tyler's age, we write the first equation above as $M = 2T$ and substitute this value of M into the second equation:

$$T + 4 = \frac{2}{3}(2T + 4) \implies 3(T + 4) = 2(2T + 4)$$

$$\implies 3T + 12 = 4T + 8 \implies T = \boxed{4}$$

Problem 10

Quadrilateral $ABCD$ is congruent to quadrilateral $WXYZ$. We are given that $\|\overline{AB}\| = 5\,\text{cm}$, $\|\overline{BC}\| = 7\,\text{cm}$, $\|\overline{YZ}\| = 6\,\text{cm}$, and $\|\overline{ZW}\| = 4\,\text{cm}$. What is the perimeter of quadrilateral $ABCD$?



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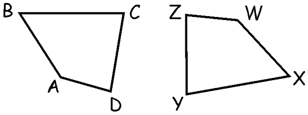

First, we list the congruent line segments:

$$\overline{XY} \equiv \overline{BC}, \quad \overline{WZ} \equiv \overline{AD}, \quad \overline{YZ} \equiv \overline{CD}, \quad \overline{WX} \equiv \overline{AB}$$

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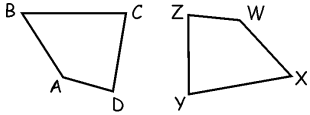

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So now, finding the perimeter of $ABCD$ is easy:

$$\begin{aligned} \text{Perimeter of } ABCD &= \|\overline{AB}\| + \|\overline{BC}\| + \|\overline{CD}\| + \|\overline{DA}\| \\ &= 5 + 7 + \|\overline{YZ}\| + \|\overline{ZW}\| \\ &= 5 + 7 + 6 + 4 = \boxed{22\,\text{cm}} \end{aligned}$$