# Warm-Up 1 Solutions

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## Problem 1

Cara has 12 more dolls than Mara, Mara has twice the number of dolls as Sara, and Sara has 15 dolls. How many dolls do the three girls have together?

$$C = M + 12, \quad M = 2S, \quad S = 15$$

Just work backwards:

$$M = 2S = 30, \quad C = M + 12 = 30 + 12 = 42$$

The answer:

$$C + M + S = 42 + 30 + 15 = \boxed{87}$$

# Problem 2

On their summer vacation, the Charen family flew from New York City to Raleigh and then to Atlanta on their way to New Orleans. Returning from New Orleans, they flew to St. Louis and then to Detroit on their way to New York City. According to the mileage chart below, what was the total length of the family's round-trip?

**Distance Between Cities (miles)**

| | Atlanta | Detroit | N.O. | NYC | Raleigh | St. Louis |
|---|---|---|---|---|---|---|
| Atlanta | - | 732 | 466 | 866 | 400 | 547 |
| Detroit | 732 | - | 1077 | 606 | 707 | 542 |
| New Orleans | 466 | 1077 | - | 1300 | 869 | 674 |
| New York City | 866 | 606 | 1300 | - | 496 | 938 |
| Raleigh | 400 | 707 | 869 | 496 | - | 828 |
| St. Louis | 547 | 542 | 674 | 938 | 828 | - |

## Problem 2

Just write down the distance for each leg obtained from the table:

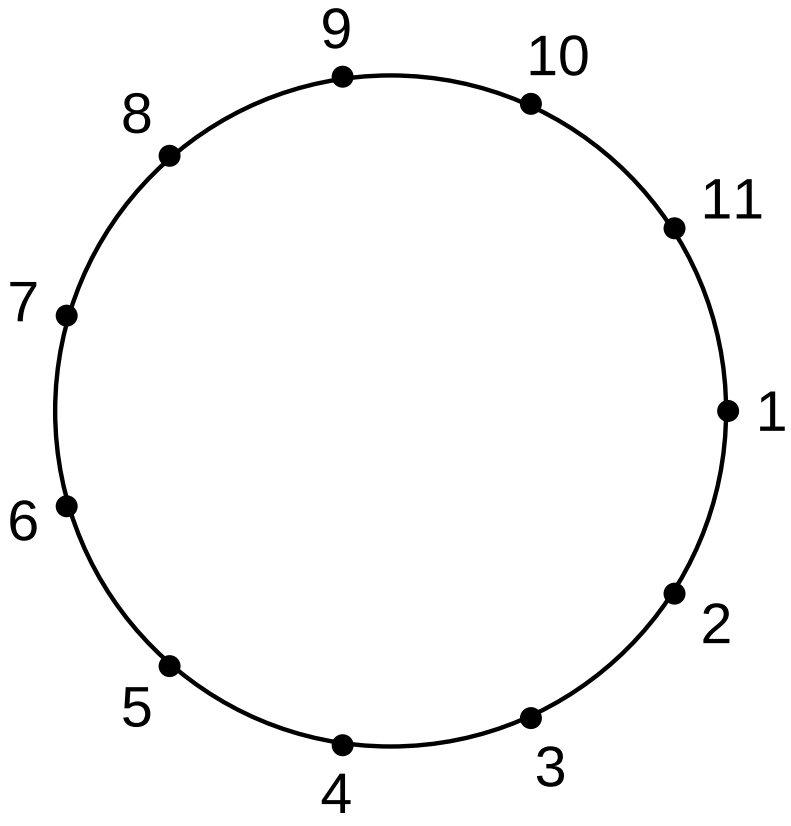
| **Leg** | **Distance** |
|---|---|
| New York → Raleigh | 496 |
| Raleigh → Atlanta | 400 |
| Atlanta → New Orleans | 466 |
| New Orleans → St. Louis | 674 |
| St. Louis → Detroit | 542 |
| Detroit → New York | 606 |
| **Total** | 3184 |

## Problem 3

Eleven girls are standing around a circle. A ball is thrown clockwise around the circle. The first girl, Ami, starts with the ball, skips the next three girls and throws to the fifth girl, who then skips the next three girls and throws the ball to the ninth girl. If the throwing pattern continues, including Ami's initial throw, how many total throws are necessary for the ball to return to Ami?

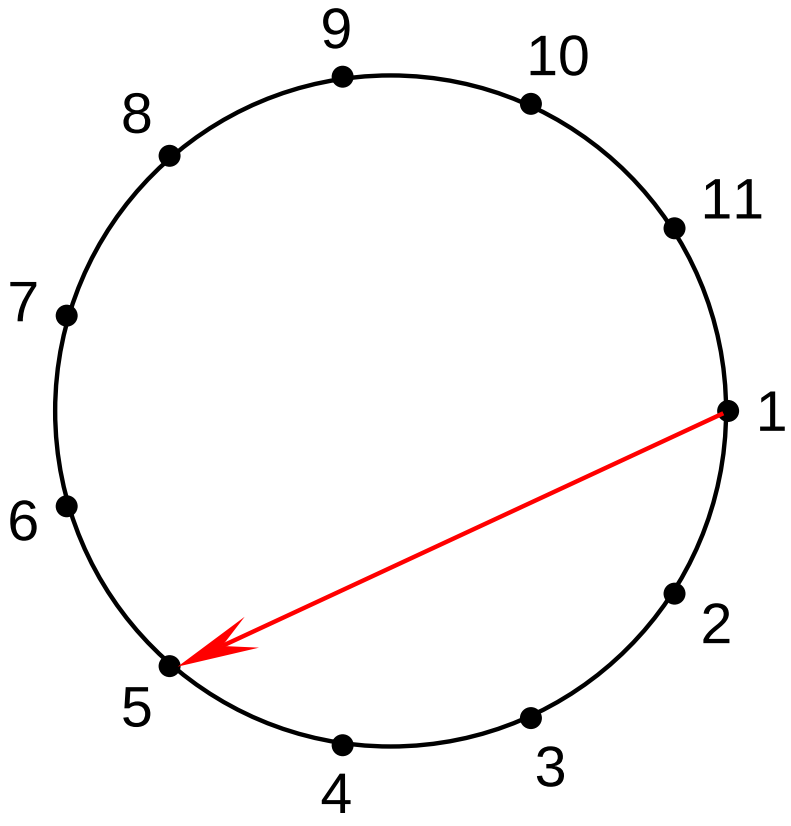
## Problem 3, Continued

A picture can really help to understand what's going on here. . .



## Problem 3, Continued

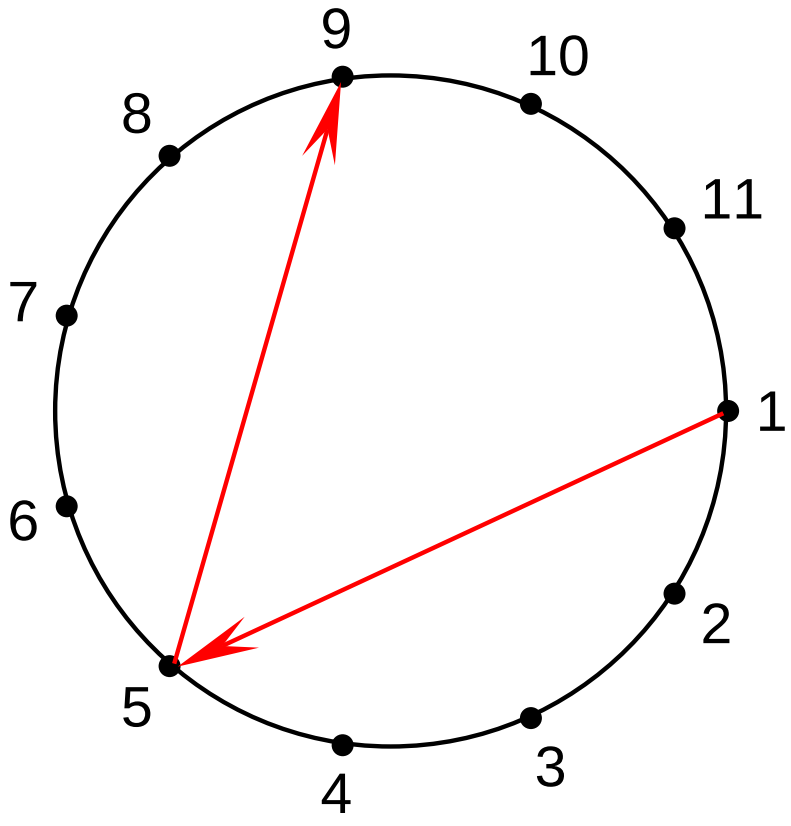
A picture can really help to understand what's going on here. . .




Path of Ball: $1 \longrightarrow 5$

## Problem 3, Continued

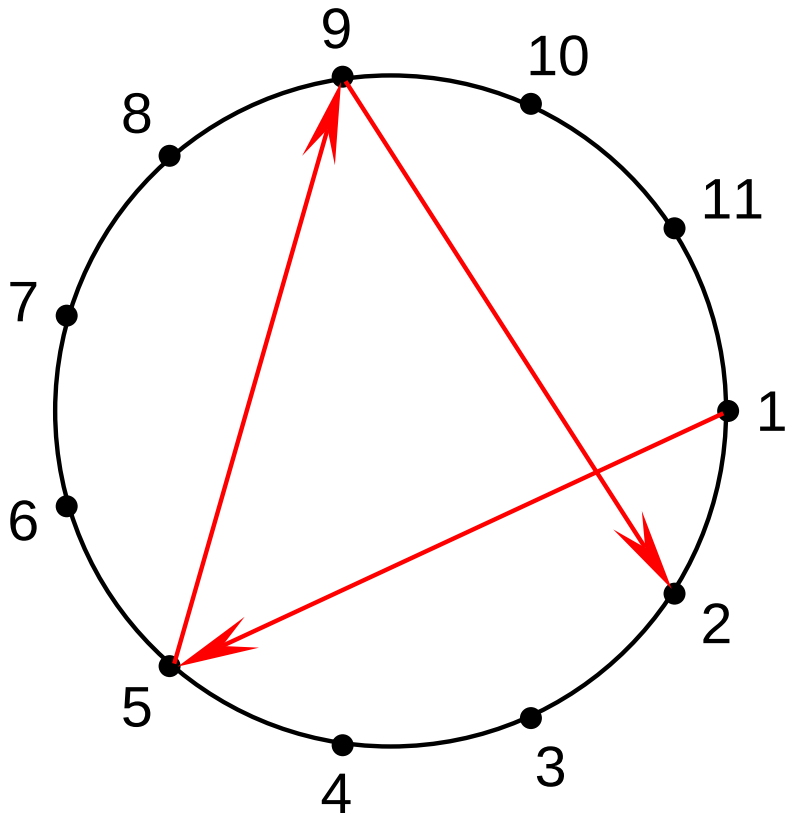
A picture can really help to understand what's going on here. . .




Path of Ball: $1 \longrightarrow 5 \longrightarrow 9$

## Problem 3, Continued

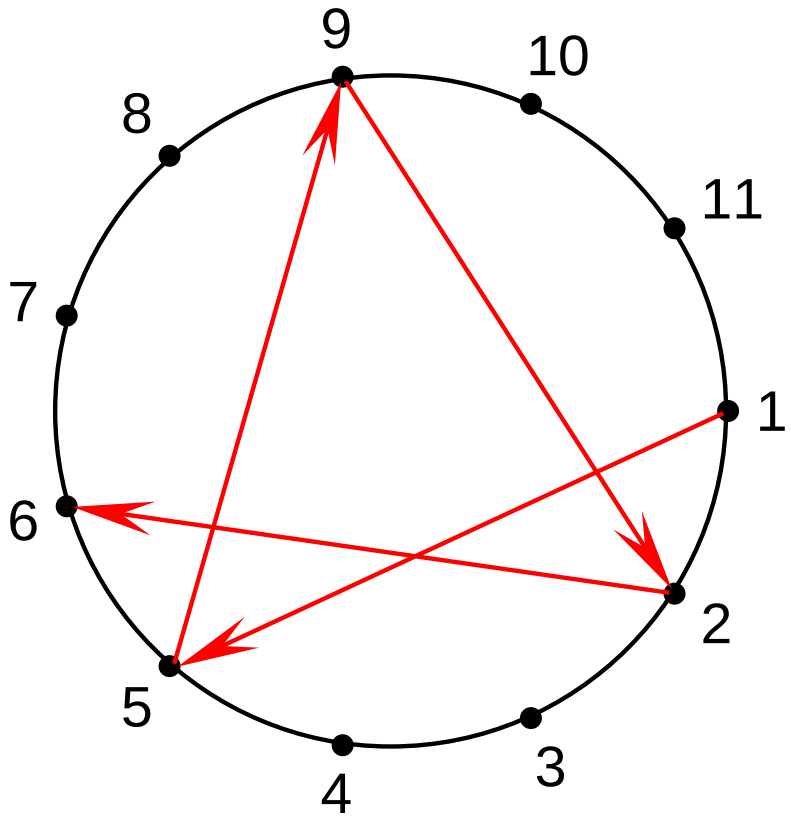
A picture can really help to understand what's going on here. . .




Path of Ball: $1 \longrightarrow 5 \longrightarrow 9 \longrightarrow 2$

## Problem 3, Continued

A picture can really help to understand what's going on here. . .




Path of Ball: $1 \to 5 \to 9 \to 2 \to 6$ Is there a pattern?

## Problem 3, Continued

The pattern is: Add 1 to the previous number in the list. If you get a number greater than 11, subtract 11.

| Toss | Ending Position |
|---|---|
| 1 | 5 |
| 2 | 9 |
| 3 | 13 – 11 = 2 |
| 4 | 6 |
| 5 | 10 |
| 6 | 14 – 11 = 3 |
| 7 | 7 |
| 8 | 11 |
| 9 | 15 – 11 = 4 |
| 10 | 8 |
| 11 | 12 – 11 = 1 |

So it takes $\boxed{11}$ throws to return to the start.

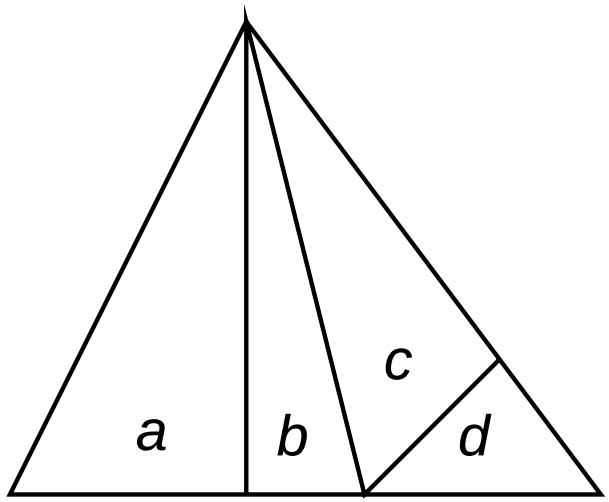
## Problem 4

Marty is 64 inches tall, and Phillip is 68 inches tall. What is the arithmetic mean of the two heights?

$$\text{Mean} = \frac{64 + 68}{2} = \boxed{66}$$

# Problem 5

How many triangles are in the figure to the right?




| Description | List | Count |
|---|---|---|
| One-region | *a*, *b*, *c*, *d* | 4 |
| Two-region | *ab*, *cd* | 2 |
| Three-region | *bcd* | 1 |
| Four-region | *abcd* | 1 |
| **Total** | | 8 |

## Problem 6

When you sketch a figure of an adult male, it is recommended that the head be $1/8$ of the total height. If the full drawn height of a 6-foot man is 10 inches, how many inches should the drawing of his head be from top to bottom? Express your answer as a decimal to the nearest hundredth.

$$\text{Head height} = \frac{1}{8} \times 10 = \boxed{1.25}$$

## Problem 7

How many multiples of 7 are between 30 and 790?

Multiples of 7 in the desired range:

$$35,\ 42,\ 49,\ \ldots,\ 770,\ 777,\ 784$$

How many intervals are there?

$$\text{Number of intervals} = \frac{784 - 35}{7} = 107$$

$$\text{Number of points} = 1 + \text{Number of intervals} = \boxed{108}$$

## Problem 8

Five aluminum cans can be recycled to make a new can. How many new cans can eventually be made from 125 aluminum cans? (Remember that the first new cans that are made can then be recycled into even newer cans!)

The first level of new cans consists of $125/5 = 25$ cans.
The second level of recycling consists of $25/5 = 5$ cans.
The third level of recycling consists of $5/5 = 1$ can.
So, the total number of new cans is

$$25 + 5 + 1 = \boxed{31}$$

## Problem 9

I am 13 years old, and my coach is 31 years old, which is my age with the digits reversed. What is the fewest number of years in which the digits of our ages will be reversed again?

Notice that my coach is 31 – 13 = 18 years older than me.

## Problem 9

One way to solve this is to use a table:

| Years | Coach Age | My Age |
|---|---|---|
| 0 | 31 | 13 |
| 1 | 32 | 14 |
| 2 | 33 | 15 |
| 3 | 34 | 16 |
| 4 | 35 | 17 |
| 5 | 36 | 18 |
| 6 | 37 | 19 |
| 7 | 38 | 20 |
| 8 | 39 | 21 |
| 9 | 40 | 22 |
| 10 | 41 | 23 |
| 11 | 42 | 24 |

So, the answer is $\boxed{11}$ years.

## Problem 9: Method 2

Let $a$ be the tens digit of the coaches age, and $b$ be the ones digit. So the coach's age will be $10a + b$ and my age is will be $10b + a$ whenever our digits are reversed. Then

$$\begin{aligned} 18 &= (\text{Coach age}) - (\text{My age}) = (10a + b) - (10b + a) \\ &= 9a - 9b = 9(a - b) \end{aligned}$$

so that $a - b = 2$. Possible coach ages are then $ab = 20$, $ab = 31$, $ab = 42$, which is the first occurrence after 31. So, the number of elapsed years is

$$42 - 31 = \boxed{11}$$

## Problem 10

How many times does the digit 6 appear in the list of all integers from 1 to 100?

Make a table:

| **Range** | **# 6s** |
|---|---|
| 1–10 | 1 |
| 11–20 | 1 |
| 21–59 | 4 |
| 60–69 | 11 |
| 70–100 | 3 |
| **Total** | 20 |