Lecture 3: Geometry I

LPSS MATHCOUNTS

28 April 2004

Oral Discussion Topics

- History of Geometry
- Points, Lines, Circles
- Definition of Angle
- Definition of π.
- Angle measures: degrees and radians

Euclid

- Born ca 325 BC, Died ca 265 BC (Alexandria, Egypt).
- The most prominent mathematician of antiquity.
- Wrote geometric treatise *The Elements*
  - Long lasting nature (more than 2000 years) of *The Elements* must make Euclid the leading mathematics teacher of all time!
- Quotations by Euclid
  - "There is no royal road to geometry."
  - A youth who had begun to read geometry with Euclid, when he had learned the first proposition, inquired, "What do I get by learning these things?" So Euclid called a slave and said "Give him threepence, since he must make a gain out of what he learns."

Similarity and Congruence

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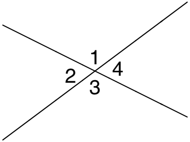
Note that if two figures are congruent, then they are automatically similar. The converse is not true.
(Sketch some figures on the board. Discuss congruence and similarity.)

Intersecting Lines



Vertical Angles Two angles whose sides form two pairs of opposite rays are called vertical. $\angle 1$ and $\angle 3$ are vertical angles.
Theorem: Vertical angles are congruent: $\angle 1 \cong \angle 3$ or $m\angle 1 = m\angle 3$.

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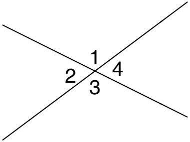

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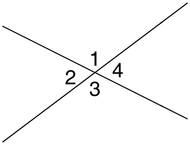

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Supplementary Angles are angles whose measures add up to 180° or π radians. Each angle is said to be the supplement of the other. $\angle 1$ and $\angle 2$ form a pair of supplementary angles.

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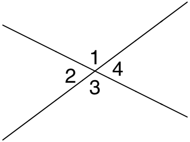

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Complementary Angles are angles whose measures add up to 90° or $\pi/2$ radians. Each angle is said to be the complement of the other.

Intersecting Lines: Example

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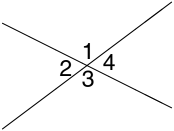
2. Find the measure of a supplement of $\angle A$.

   Solution: Let $\angle C$ be the supplement of $\angle A$. Then $m\angle A + m\angle C = 180^\circ$, so that $m\angle C = 180^\circ - m\angle A = 180^\circ - 20^\circ = 160^\circ$.

Intersecting Lines: Example



- Assume $m\angle 3 = 120^\circ$. Find the measures of the three other angles.

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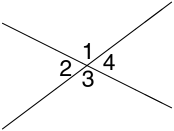

- Assume $m\angle 3 = 120^{\circ}$. Find the measures of the three other angles.
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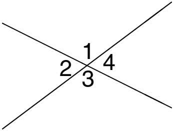

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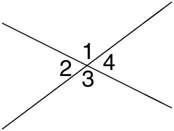

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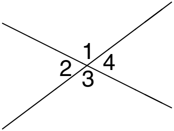

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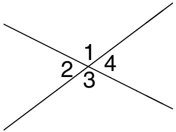

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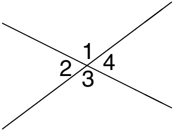

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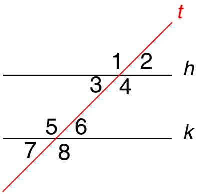
Parallel Lines with a Transversal



$h \parallel k$, with transversal t.

Interior angles: $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$.

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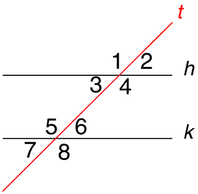

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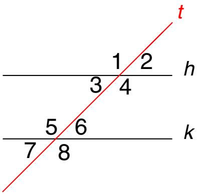

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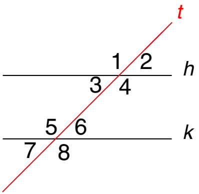

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Same-side Interior Angles are two interior angles on the same side of the transversal: ($\angle 3, \angle 5$) and ($\angle 4, \angle 6$).

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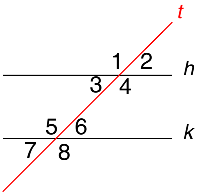

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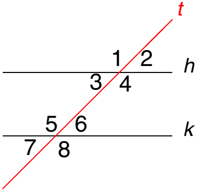

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Postulate: If $h \parallel k$, then corresponding angles are congruent.

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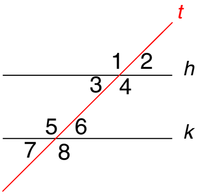

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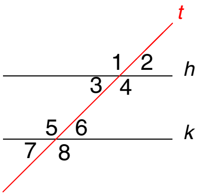
Theorem: If $h \parallel k$, then alternate interior angles are congruent.

Parallel Lines with Transversal Example



$h \parallel k$, with transversal t.

- Assume $m\angle 1 = 135^\circ$. Find all other labeled angles:

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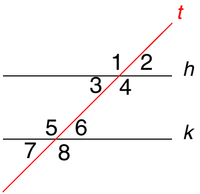

h || *k*, with transversal *t*.

- Assume $m\angle 1 = 135^\circ$. Find all other labeled angles:
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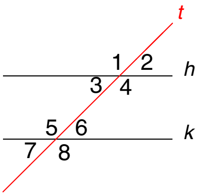
Parallel Lines with Transversal Example



$h \parallel k$, with transversal t.

- Assume $m\angle 1 = 135^\circ$. Find all other labeled angles:
- $m\angle 2 = 180^\circ - m\angle 1 = 180^\circ - 135^\circ = 45^\circ$

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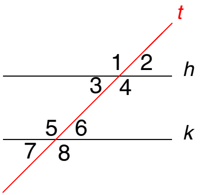

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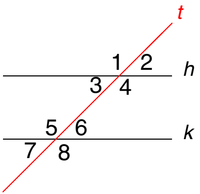

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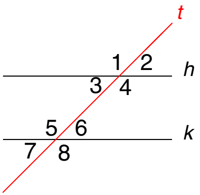

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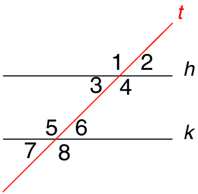

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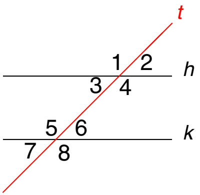

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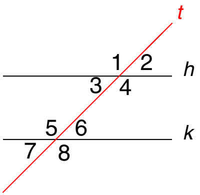

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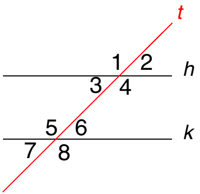

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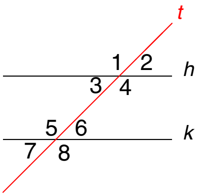

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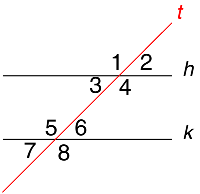

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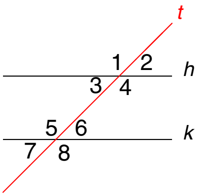

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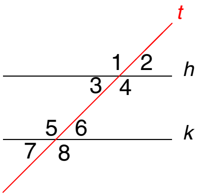

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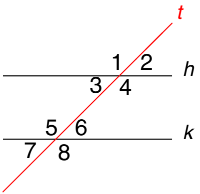

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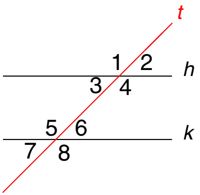
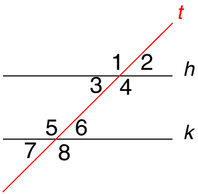


$h \parallel k$, with transversal t.

- What is the value of the sum $m\angle 3 + m\angle 5$?

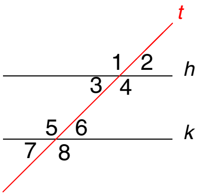
Parallel Lines with Transversal Example



$h \parallel k$, with transversal t.

- What is the value of the sum $m\angle 3 + m\angle 5$?
- What is the value of the sum $m\angle 4 + m\angle 6$?

Parallel Lines with Transversal Example



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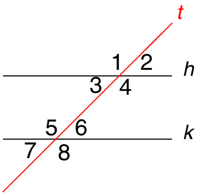
- What can you conclude about same-side interior angles?

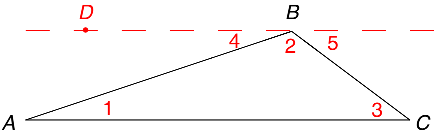
Parallel Lines with Transversal Example



$h \parallel k$, with transversal t.

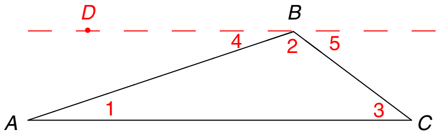
- What is the value of the sum $m\angle 3 + m\angle 5$?
- What is the value of the sum $m\angle 4 + m\angle 6$?
- What can you conclude about same-side interior angles?
- Same-side interior angles are supplementary

Sum of Interior Angles of a Triangle



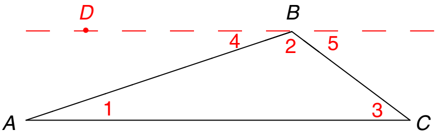
1. Through B, draw $\overline{DB}$ parallel to $\overline{AC}$.

Sum of Interior Angles of a Triangle



1. Through B, draw $\overline{DB}$ parallel to $\overline{AC}$.
2. Note that $m\angle 4 + m\angle 2 + m\angle 5 = 180^\circ$.

Sum of Interior Angles of a Triangle



1. Through B, draw $\overline{DB}$ parallel to $\overline{AC}$.
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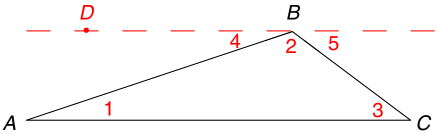
3. Note that $\angle 1 \cong \angle 4$, and $\angle 5 \cong \angle 3$. (Why?)

Sum of Interior Angles of a Triangle



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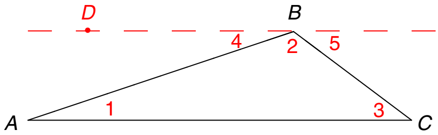
4. Therefore, $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$.

Sum of Interior Angles of a Triangle



1. Through B, draw $\overline{DB}$ parallel to $\overline{AC}$.
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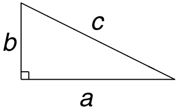
The sum of the interior angles of a triangle equals 180°.

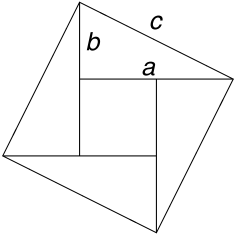
Proof of Pythagorean Theorem



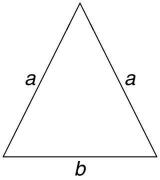
Area of triangle = $\frac{1}{2}ab$

Consider four copies of the triangle, rotated by 0°, 90°, 180°, and 270°. The side length of the small, central square is $a - b$. The area of the large, outer square of side length c can be calculated two ways:

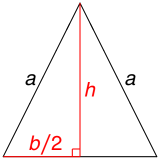
$$c^2 = 4\left(\frac{1}{2}ab\right) + (a-b)^2 = 2ab + a^2 - 2ab + b^2 = a^2 + b^2$$



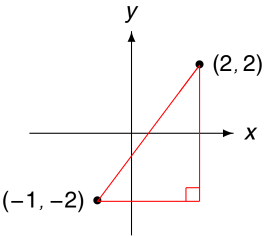
Height and Area of Isosceles Triangle



1. Draw perpendicular bisector from base to opposite vertex.
2. From Pythagoras: $h^2 + (b/2)^2 = a^2$ so that
3. $h = \sqrt{a^2 - (b/2)^2}$.
4. The area A is then found using $A = \frac{1}{2}bh$.

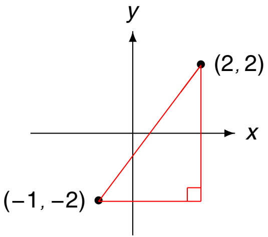


Distance Between Points in the Plane



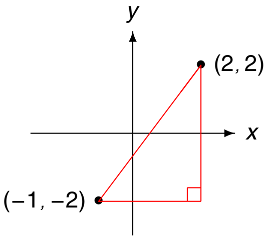
1. Distance between points measured parallel to x-axis is

Distance Between Points in the Plane



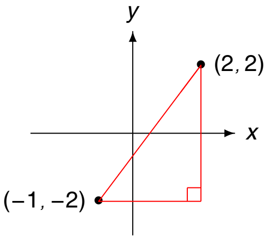
1. Distance between points measured parallel to x-axis is
   $X = 2 - (-1) = 3$.

Distance Between Points in the Plane



1. Distance between points measured parallel to x-axis is $X = 2 - (-1) = 3$.
2. Distance between points measured parallel to y-axis is

Distance Between Points in the Plane



1. Distance between points measured parallel to x-axis is $X = 2 - (-1) = 3$.
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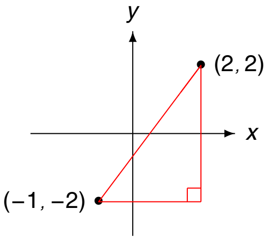
Distance Between Points in the Plane



1. Distance between points measured parallel to *x*-axis is
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3. Distance between points is equal to length of hypotenuse which is

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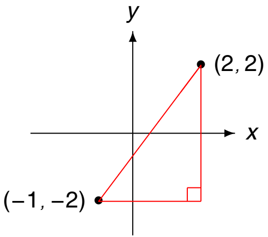

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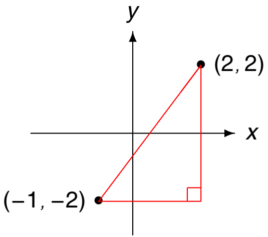
   $d = \sqrt{X^2 + Y^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.

Distance Between Points in the Plane



1. Distance between points measured parallel to x-axis is $X = 2 - (-1) = 3$.
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3. Distance between points is equal to length of hypotenuse which is $d = \sqrt{X^2 + Y^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.
4. For general case of point $A : (x_A, y_A)$ and point $B : (x_B, y_B)$,
$$\text{distance}(A, B) = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$