Arithmetic, Algebra, Number Theory

Peter Simon

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- **Real Numbers** The set of numbers containing both the rationals and irrationals.

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$x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$	$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = 4$

Los Primeros MATHCOUNTS

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(Answers should be simplified as much as possible)

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MATHCOUNTS

Generating or Testing for Prime Numbers

Trial Division To see if an individual (small) integer is prime, trial division works well: just divide by all the primes less than (or equal to) its square root. For example, to show 211 is prime, just divide by 2, 3, 5, 7, 11, and 13. Since none of these divides the number evenly, it is a prime.

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Sieve of Eratosthenes If we wish to generate a list of consecutive prime numbers less than some given number, the elimination technique called the sieve of Eratosthenes is faster.

Prime Numbers—The Sieve of Eratosthenes

Recall that the prime numbers are natural numbers having exactly two factors (the number itself and 1). The most efficient way to find all of the small primes (say all those less than 10,000,000) is by using the Sieve of Eratosthenes (ca 240 BC):

1. Start with a list of natural numbers from 2 to, say, 37.
   2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37

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2. The first number 2 is prime. Retain it, but cross out all multiples of 2 in the list.
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3. The first remaining number 3 is prime. Retain it, but cross out all multiples of 3 in the list.
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Prime Numbers—The Sieve of Eratosthenes (Cont.)

4. The first remaining number 5 is prime. Retain it, but cross out all multiples of 5 in the list.
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5. The first remaining number 7 is prime. Retain it, but cross out all multiples of 7 in the list.

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6. The first remaining number is $11 > \sqrt{37}$ so we do not need to check for multiples of it or any of the other remaining numbers in the list (why?). All the remaining numbers in the list are primes: $2, 3, 5, 7, 11, 13, 17, 23, 29, 31, 37$

GCF: Greatest Common Factor

The greatest common factor (GCF) of two natural numbers is the greatest factor that divides both of the numbers.

Examples: $\text{GCF}(2, 3) = 1$, $\quad \text{GCF}(2, 4) = 4$, $\quad \text{GCF}(10, 15) = 5$.

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- Prime factors of 90: $2^1 \times 3^2 \times 5^1$
- Prime factors of 24: $2^3 \times 3^1 \times 5^0$
- Therefore, $GCF(90, 24) = 2^1 \times 3^1 \times 5^0 = 6$.

LCM: Least Common Multiple

A common multiple is a number that is a multiple of two or more numbers. The common multiples of 3 and 4 are $0, 12, 24, \ldots$ The least common multiple (LCM) of two numbers is the smallest number (not zero) that is a multiple of both. So $\text{LCM}(3, 4) = 12$.

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- Prime factors of 90: $2^1 \times 3^2 \times 5^1$
- Prime factors of 24: $2^3 \times 3^1 \times 5^0$
- Therefore, $\text{LCM}(90, 24) = 2^3 \times 3^2 \times 5^1 = 360.$

Product of GCF and LCM

Note that

$$\text{GCF}(90, 24) \times \text{LCM}(90, 24) = 6 \times 360 = 2160 = 90 \times 24$$

This is true in general:

$$\boxed{\text{GCF}(a, b) \times \text{LCM}(a, b) = a \times b}$$

The product of the GCF and LCM of two natural numbers is equal to the product of the numbers themselves.

Repeating Decimals

Example: $x = 0.32\overline{49} \equiv 0.32494949\ldots$

To convert the repeating decimal form to an equivalent fraction:

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$$100x = 32.\overline{49}$$

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$$9900x = 3217$$

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To convert the repeating decimal form to an equivalent fraction:

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$$100x = 32.\overline{49}$$

2. Multiply by a power of 10 that matches the length of the repeating part:

$$100 \times 100x = 10000x = 3249.\overline{49}$$

3. Subtract the two equations obtained above:

$$10000x - 100x = 3249.\overline{49} - 32.\overline{49}$$

$$9900x = 3217$$

$$x = \frac{3217}{9900}$$

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Some Common Repeating Decimals

$$0.\overline{1} = \frac{1}{9}$$

$$0.\overline{2} = \frac{2}{9}$$

$$0.\overline{3} = \frac{3}{9} = \frac{1}{3}$$

$$0.\overline{4} = \frac{4}{9}$$

$$0.\overline{5} = \frac{5}{9}$$

$$0.\overline{6} = \frac{6}{9} = \frac{2}{3}$$

$$0.\overline{7} = \frac{7}{9}$$

$$0.\overline{8} = \frac{8}{9}$$

$$0.\overline{9} = \frac{9}{9} = 1$$

Counting the Number of Factors

- **Prime Numbers**

$$7 = 1 \times 7 \quad \text{(2 factors)}$$

$$11 = 1 \times 11 \quad \text{(2 factors)}$$

Prime numbers have exactly 2 factors

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- **Prime Number Raised to Integer Power**

$$9 = 3^2 \text{ has 3 factors: } 1, 3, 9$$

$$27 = 3^3 \text{ has 4 factors: } 1, 3, 9, 27$$

A prime raised to a positive integer power n has $n + 1$ factors.

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- **Product of Two Distinct Primes**

$$21 = 3 \times 7, \quad \text{4 factors: } 1, 3, 7, 21$$

A product of two distinct primes has exactly 4 factors

Counting the Number of Factors (Cont.)

- **Product of Prime Numbers Raised to Powers**

$100 = 4 \times 25 = 2^2 \times 5^2$ has 9 factors: $1, 2, 4, 5, 10, 20, 25, 50, 100$

Note that $9 = 3 \times 3 = (2 + 1) \times (2 + 1)$

$72 = 8 \times 9 = 2^3 \times 3^2$ has 12 factors: $1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72$

Note that $12 = 4 \times 3 = (3 + 1) \times (2 + 1)$

Counting the Number of Factors (Cont.)

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- Let $P_1, P_2, P_3, \ldots$ be prime numbers and let $n_1, n_2, n_3, \ldots$ be natural numbers. Then the number of factors of

  $$P_1^{n_1} \times P_2^{n_2} \times P_3^{n_3} \times \cdots$$

  is

  $$(n_1 + 1) \times (n_2 + 1) \times (n_3 + 1) \times \cdots$$

Counting the Number of Factors (Cont.)

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  200

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$$200 = 2^3 \times 5^2 \implies (3+1)(2+1) = 4 \times 3 = 12 \text{ factors}$$