Name:_____

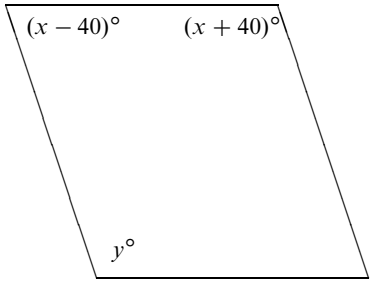
Los Primeros MATHCOUNTS 2004–2005
Homework 3
May 12, 2004

1. Recall that the sum of the interior angles of a triangle is $180°$. Use this fact to help fill out the table below, by first sketching a polygon, then connecting one vertex to each of the others, thus dividing the figure into triangles. Multiply the number of triangles so obtained by $180°$ to find the sum of the interior angles.

Regular Polygons

# Sides	# Triangles	Sum of Interior Angles	Measure of Single Interior Angle
3	$3-2=1$	$1\times 180°=180°$	$180°/3=60°$
4	$4-2=2$	$2\times 180°=360°$	$360°/4=90°$
5			
6			
N			

2. What do x and y have to be to make this figure a parallelogram?



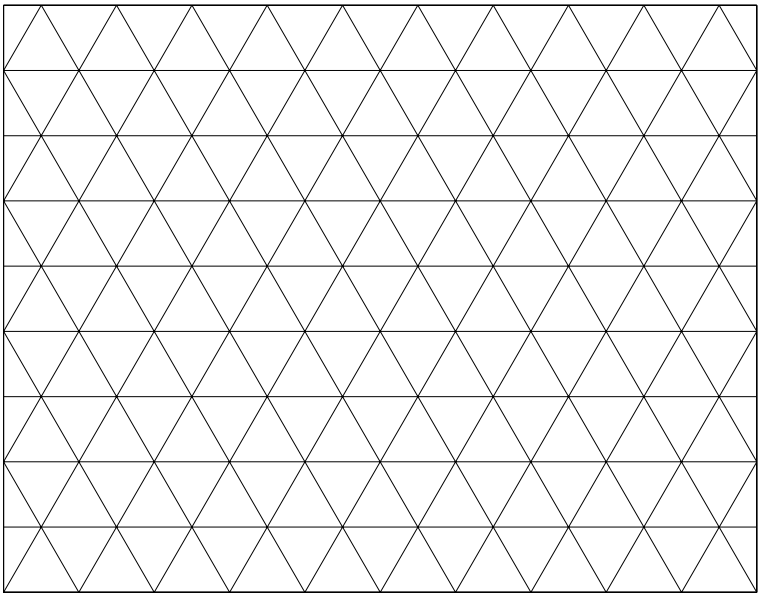
3. Suppose that a and b are distinct, prime numbers. How many factors does the number $8ab$ have?

4. How many centimeters are in the length of the longest side of a rectangle whose area is 108 square centimeters and whose perimeter is 42 centimeters?

5. Outline and/or shade-in the following figures on the triangle diagram below using only the existing lines:

a) equlateral triangle
b) isosceles triangle (non-equilateral)
c) right triangle
d) scalene triangle
e) parallelogram (not a rectangle or rhombus)
f) rectangle
g) trapezoid (non-isosceles)
h) isosceles trapezoid
i) pentagon
j) hexagon
k) heptagon
l) octagon

(Suggestion: Work in pencil!)



Definitions

Circle The set of points in the plane a given distance from a given point.

Congruent Equal measure.

Hexagon A six-sided polygon.

Heptagon A seven-sided polygon.

Octagon An eight-sided polygon.

Parallelogram A quadrilateral with both pairs of opposite sides parallel.

Pentagon A five-sided polygon.

Polygon A plane figure with straight sides.

Quadrilateral A four-sided polygon.

Rectangle A quadrilateral with four right angles; it is a special kind of parallelogram.

Rhombus A parallelogram with all four sides congruent.

Similar Figures whose corresponding angles are equal; figures of the same shape but not necessarily the same size.

Trapezoid A quadrilateral with exactly one pair of opposite sides parallel.

-Isosceles A trapezoid with the un-parallel sides congruent.

Triangle A three-sided polygon.

-Equilateral A triangle with all sides congruent.

-Isosceles A triangle with at least two sides congruent.

-Right A triangle with a right angle.

-Scalene A triangle with no sides congruent.

Los Primeros MATHCOUNTS 2004–2005
Answer Key for Homework 3
May 12, 2004

1. Recall that the sum of the interior angles of a triangle is $180°$. Use this fact to help fill out the table below, by first sketching a polygon, then connecting one vertex to each of the others, thus dividing the figure into triangles. Multiply the number of triangles so obtained by $180°$ to find the sum of the interior angles.

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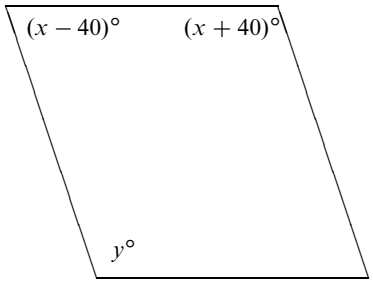
5	$5-2=3$	$3\times 180°=540°$	$108°$
6	$6-2=4$	$4\times 180°=720°$	$120°$
N	$N-2$	$(N-2)\times 180°$	$\frac{N-2}{N}\times 180°$

2. What do x and y have to be to make this figure a parallelogram?



Answer: If the figure is to be a parallelogram, then it must be true that each pair of adjacent vertices defines a pair of same-side interior angles. Thus, we must have that

$$(x-40°)+(x+40°)=180°$$

which is easily solved to find that $x=90°$. Similarly, we must have

$$(x-40°)+y=180° \implies 50°+y=180°$$

so that $y=130°$.

3. Suppose that a and b are distinct, prime numbers. How many factors does the number $8ab$ have?

Answer: We know that for any number written as a product of distinct prime numbers raised to powers, the number of factors can be found by adding 1 to each exponent and then multiplying. Thus, since $8ab=2^3a^1b^1$ we have # Factors $=(3+1)(1+1)(1+1)=(4)(2)(2)=16$.

4. How many centimeters are in the length of the longest side of a rectangle whose area is 108 square centimeters and whose perimeter is 42 centimeters?

Answer: The perimeter of a rectangle is twice the sum of the length and width, thus the length plus the width of the rectangle is $42\div 2=21$ centimeters. So, $L+W=21$ and $LW=108$. Algebra or trial and error gives $L=12$, $W=9$ so the length is **12 cm.**

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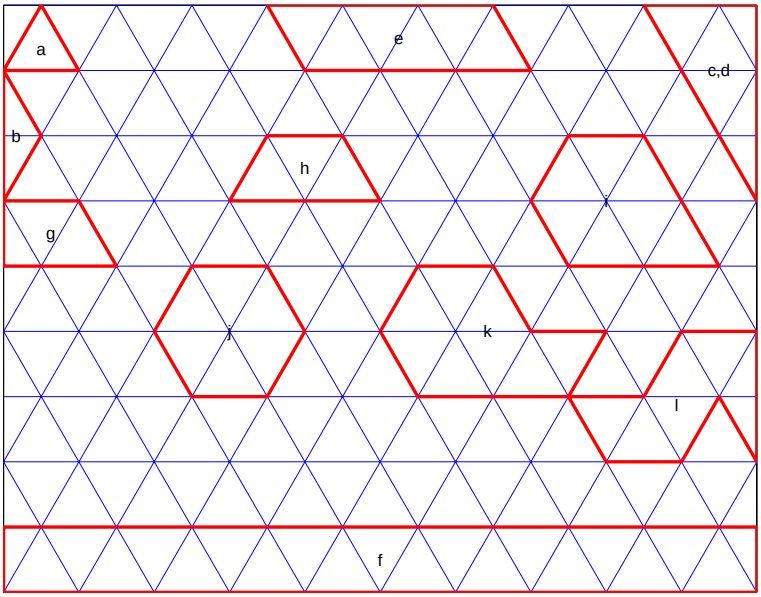
k) heptagon
l) octagon

One of many possible solutions:



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