Name:_____

Los Primeros MATHCOUNTS 2004–2005
Homework 2
April 21, 2004

1. What is the smallest positive integer that has 8, 30, and 54 as factors?

2. Solve $2^2 \times 4^2 \times 8^2 \times 16^2 \times \cdots \times 1024^2 = 2^x$ for x.

3. Use the Sieve of Eratosthenes to find all the prime numbers less than 100.

4. There are two clocks on the wall. The first clock runs fast, taking 54 minutes for the minute hand to make one revolution. The second clock runs slow, taking 63 minutes to make one revolution. If both clocks start with their minute hands pointing straight up, how many minutes will elapse before they again simultaneously point straight up?

Los Primeros MATHCOUNTS 2004–2005
Answer Key for Homework 2
April 21, 2004

1. What is the smallest positive integer that has 8, 30, and 54 as factors?

   Answer: We are being asked for the LCM of 8, 30, and 54. Since

   $$\begin{aligned} 8 &= 2^3 \\ 30 &= 2 \times 3 \times 5 \\ 54 &= 2 \times 3^3 \end{aligned}$$

   then the LCM is $2^3 \times 3^3 \times 5 = 1080$.

2. Solve $2^2 \times 4^2 \times 8^2 \times 16^2 \times \cdots \times 1024^2 = 2^x$ for x.

   Answer: We can rewrite the given equation as

   $$\begin{aligned} 2^x &= 2^2 \times (2^2)^2 \times (2^3)^2 \times (2^4)^2 \times \cdots \times (2^{10})^2 \\ &= 2^2 \times 2^{2\times 2} \times 2^{3\times 2} \times 2^{4\times 2} \times \cdots \times 2^{10\times 2} \\ &= 2^{2+4+6+8+\cdots+20} \\ &= 2^{110} \end{aligned}$$

   so that $x = 110$.

3. Use the Sieve of Eratosthenes to find all the prime numbers less than 100.

   Answer: The numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

4. There are two clocks on the wall. The first clock runs fast, taking 54 minutes for the minute hand to make one revolution. The second clock runs slow, taking 63 minutes to make one revolution. If both clocks start with their minute hands pointing straight up, how many minutes will elapse before they again simultaneously point straight up?

   Answer: The answer is the LCM of 54 and 63 (do you see why?). Since

   $$\begin{aligned} 54 &= 2 \times 3^3, \\ 63 &= 3^2 \times 7, \end{aligned}$$

   Then the LCM is $2 \times 3^3 \times 7 = 378$. Thus 378 minutes (approximately 6.3 hours) are required.