Name:____

Los Primeros MATHCOUNTS 2004–2005
Homework 1
April 6, 2004

1. Find the sum of all odd numbers between 0 and 1000.

2. Find the sum of all even numbers between 0 and 1000, inclusive.

3. The squares of a chessboard are numbered as shown in the following table:

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
			⋮				
57	58	59	60	61	62	63	64

   One grain of rice is laid in square 1, two in square 2, three in square 3, four in square 4, etc. How many grains are there altogether in the last five rows?

4. When three integers are added two at a time, three distinct sums are obtained: 32, 48, and 46. What is the sum of the three integers?

5. Each term in a sequence is the sum of the previous two terms. If the sequence contains the terms a, b, c, 12, 19, and 31, in that order, what is the value of a?

Answer Key for Homework 1

1. Find the sum of all odd numbers between 0 and 1000.

   Answer: This is an arithmetic series: $S = 1+3+5+\cdots+999$. We know that the sum is equal to the average of the first and last term multiplied by the number of terms. By comparing a given term to its place in the list, we see that the first term is $1 = 2\times1-1$, the second term is $3 = 2\times3-1$, the third term is $5 = 2\times3-1$ and so on. Thus, the 500th term is $2\times500-1 = 999$ so that there must be 500 terms in total. The sum is therefore

   $$S = \frac{1+999}{2}\times 500 = 500\times 500 = 250,000.$$

2. Find the sum of all even numbers between 0 and 1000, inclusive.

   Answer: This arithmetic series is summed in the same way as in the previous problem. The sum is $S = 2+4+6+\cdots+1000$ and there are 500 terms. Thus,

   $$S = \frac{2+1000}{2}\times 500 = 501\times 500 = 250,500.$$

3. The squares of a chessboard are numbered as shown in the following table:

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
			⋮				
57	58	59	60	61	62	63	64

   One grain of rice is laid in square 1, two in square 2, three in square 3, four in square 4, etc. How many grains are there altogether in the last five rows?

   Answer: The number we seek is $S = 25+26+27+\cdots+64$. Since this is an arithmetic series with $a = 25$, $b = 64$, and $n = 40$, the answer is

   $$S = n\frac{a+b}{2} = 40\frac{25+64}{2} = 1780$$

   Another way to look at the problem is to consider S to be the difference between the total number of grains of rice and the number located in the first three rows. Since the last square in row 3 is square #24, then

   $$S = T_{64} - T_{24} = \frac{64\times65}{2} - \frac{24\times25}{2} = 32\times65 - 12\times25 = 1780$$

4. When three integers are added two at a time, three distinct sums are obtained: 32, 48, and 46. What is the sum of the three integers?

Answer: Denote the three integers as a,b, and c. We are given that $a + b = 32$, $a+c = 48$, and $b+c = 46$. If we add all three equations together, we obtain $2(a + b + c) = 32 + 48 + 46 = 126$. Therefore, $a + b + c = 126/2 = 63$.

5. Each term in a sequence is the sum of the previous two terms. If the sequence contains the terms a, b, c, 12, 19, and 31, in that order, what is the value of a?

   Answer: Working backwards, we have that $c + 12 = 19$, so $c = 7$. But $12 = b + c = b + 7$ so $b = 5$. Finally, $c = 7 = a + b = a + 5$ so that we must have $a = 2$.