# SPACE SYSTEMS/LORAL

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| To | H. Luh | File Name | curvature.tex |
| From | P. Simon | Memo Number | AAS/01/00??/IRAD |
| Subject | Calculation of surface curvature for a surface parameterized as $z = f(x, y)$. | Distribution | |

This memo records the formulas for calculating the principal curvatures of a surface parameterized in the form $z = f(x, y)$. The formulas are taken from Example 5 on page 162 of [1].

One first computes the Gaussian curvature

$$K = \frac{f_{xx} f_{yy} - f_{xy}^2}{\left(1 + f_x^2 + f_y^2\right)^2} \tag{1}$$

and mean curvature

$$H = \frac{\left(1 + f_x^2\right) f_{yy} + \left(1 + f_y^2\right) f_{xx} - 2 f_x f_y f_{xy}}{2\left(1 + f_x^2 + f_y^2\right)^{3/2}} \tag{2}$$

The principal curvatures $k_1$ and $k_2$ are then found using

$$k_1 = H + \sqrt{H^2 - K} \tag{3}$$

$$k_2 = H - \sqrt{H^2 - K} \tag{4}$$

and the principal radii of curvature (without attached sign) are calculated as

$$r_i = |1/k_i|, \quad i = 1, 2. \tag{5}$$

The minimum of the two principal radii of curvature is the minimum radius of curvature of the surface at the given point.

## References


[1] M. P. do Carmo, *Differential Geometry of Curves and Surfaces*. New York: Prentiss-Hall, 1976.