Math League Contest #5, March 10, 1998

ACHS Math Competition Team

October 27, 2009

Problem 5-1

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Problem 5-2

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$$143\,000\,000 = 143 \times 10^6 = 143 \times 5^6 \times 2^6 = 13 \times 11 \times 5^6 \times 2^6$$

so the largest prime factor is 13.

Problem 5-3

By finding a gold nugget, a prospector is said to “strike it rich.” If, on any given day, a prospector has a probabilty of $\frac{1}{8}$ of striking it rich, what is the probability that at least one of the prospectors Al and Barb strike it rich on a day that they both mine for gold?

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Method I

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$$P = \frac{1}{8} \times \frac{7}{8}$$

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Problem 5-3 (Cont.)

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Method II

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so that the average of the three numbers is $2/3$.

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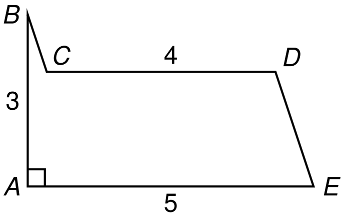
In the diagram, $\overline{AE} \perp \overline{AB}$, $\overline{CD} \parallel \overline{AE}$, and $\overline{BC} \parallel \overline{DE}$. If $CD = 4$, $AB = 3$, and $AE = 5$, and if the distance from $\overline{AE}$ to $\overline{CD}$ is 2, what is the area of pentagon $ABCDE$?



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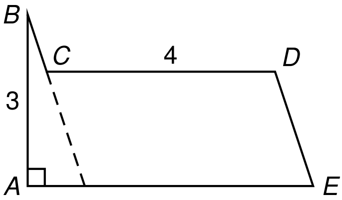

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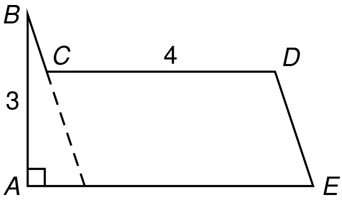

Extend $\overline{BC}$ until it intersects $\overline{AE}$, yielding a parallelogram and a right triangle.

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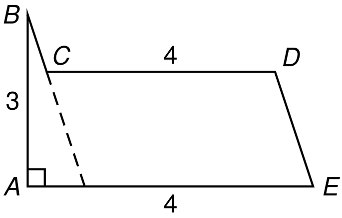

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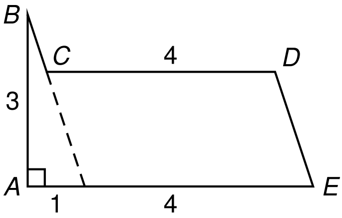

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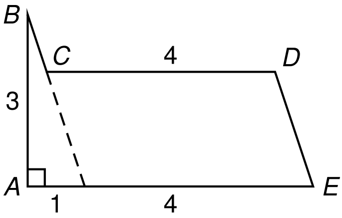

Extend $\overline{BC}$ until it intersects $\overline{AE}$, yielding a parallelogram and a right triangle.
The area of the pentagon is then

$$\frac{1}{2} \times 1 \times 3 + 2 \times 4$$

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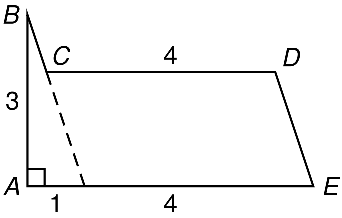

Extend $\overline{BC}$ until it intersects $\overline{AE}$, yielding a parallelogram and a right triangle.
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$$\frac{1}{2} \times 1 \times 3 + 2 \times 4 = 1.5 + 8 = 9.5.$$

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$$P(x) = n_0 + n_1 x + n_2 x^2 + \cdots + n_N x^N$$

where N is the order of the polynomial, and $n_0, n_1, \ldots, n_N$ are integers.

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Note that $P(x)$ is not linear, since $n_0 + n_1(\sqrt{3} + \sqrt{2}) \neq \sqrt{3} - \sqrt{2}$ for any integral n_0 and n_1.

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In terms of x, what is the polynomial P of least degree, with integral coefficients, for which $P(\sqrt{3}+\sqrt{2}) = \sqrt{3} - \sqrt{2}$?

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$$\begin{aligned}
\sqrt{3}-\sqrt{2} &= n_0 + n_1\left(\sqrt{3}+\sqrt{2}\right) + n_2(\sqrt{3}+\sqrt{2})^2 + n_3(\sqrt{3}+\sqrt{2})^3 \\
&= n_0 + n_1(\sqrt{3}+\sqrt{2}) + n_2(5+2\sqrt{6}) + n_3(9\sqrt{3}+11\sqrt{2}) \\
&= n_0 + 5n_2 + \sqrt{2}(n_1 + 11n_3) + \sqrt{3}(n_1 + 9n_3) + 2n_2\sqrt{6}
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We conclude $n_2 = 0$

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\end{aligned}$$

We conclude $n_2 = 0$ and therefore $n_0 = 0$.

Problem 5-6 (cont.)

$$\sqrt{3} - \sqrt{2} = \sqrt{2}(n_1 + 11n_3) + \sqrt{3}(n_1 + 9n_3)$$

Problem 5-6 (cont.)

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The integral multipliers of the radicals on each side of the equation must be the same:

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By subtracting these two equations, we find $2n_3 = -2$ or $n_3 = -1$.

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The integral multipliers of the radicals on each side of the equation must be the same:

$$-1 = n_1 + 11n_3, \quad 1 = n_1 + 9n_3$$

By subtracting these two equations, we find $2n_3 = -2$ or $n_3 = -1$. Then $n_1 = 10$.

Problem 5-6 (cont.)

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The polynomial is therefore $P(x) = 10x - x^3$.