ACHS Math Team 2007–2008

Introduction to Mathematical Logic (Proposition Calculus)

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What is Logic?

- A systematic way of assigning truth to statements.
- Based on a mathematical framework that agrees with our intuition.

Statements and Compound Statements

Statements (or verbal assertions) will be denoted by lower-case letters such as

$$p, \quad q, \quad r.$$

The fundamental property of a statement is that it is either true or false, but not both. The truthfulness of falsity of a statement is called its truth value. We can combine statements together using relational operators to form compound statements, as shown below.

Examples of Statements

1. "Roses are red and violets are blue" is a compound statement with substatements "Roses are red" and "violets are blue" connected by an "and" relation.
2. "He is very smart or he studies very hard every night" is a compound statement with substatements "He is very smart" and "he studies very hard every night" connected by an "or" relation.
3. "Where are you going?" is not a statement, since it can not be assigned a true or false value.

Logical Conjunction (AND)

Consider the statements “It is raining outside” and “Jack has blue eyes.” We can assign each statement a logical value, either true (T) or false (F). Now consider the compound statement “It is raining outside and Jack has blue eyes.” How do we assign a truth value to the compound statement? There are four cases to be considered. Suppose that

1. It *really is* raining outside and Jack *really does* have blue eyes.
2. It *really is* raining outside and Jack *really does not* have blue eyes.
3. It *really is not* raining outside and Jack *really does* have blue eyes.
4. It *really is not* raining outside and Jack *really does not* have blue eyes.

Most people would agree that the compound statement “It is raining outside and Jack has blue eyes” is true in case 1, and false in cases 2, and 3 and case 4.

Truth Table for Logical Conjunction ("AND")

Suppose p and q are statements. Then the logical conjunction $p \wedge q$ (read "p and q") is the compound statement having the following truth table:

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- The conjunction ("and") of two statements is true only when both statements are true.
- Note the similarity of the logical conjunction operator $\wedge$ to the set intersection operator $\cap$. They are closely related in meaning, also.

Logical Disjunction (OR)

Consider the statements “It is raining outside” and “Jack has blue eyes.” We can assign each statement a logical value, either true (T) or false (F). Now consider the compound statement “It is raining outside or Jack has blue eyes.” How do we assign a truth value to the compound statement? There are four cases to be considered. Suppose that

1. It *really is* raining outside and Jack *really does* have blue eyes.
2. It *really is* raining outside and Jack *really does not* have blue eyes.
3. It *really is not* raining outside and Jack *really does* have blue eyes.
4. It *really is not* raining outside and Jack *really does not* have blue eyes.

Most people would agree that the compound statement “It is raining outside or Jack has blue eyes” is true in cases 1, 2, and 3, and it is false only in case 4.

Truth Table for Logical Disjunction ("OR")

Suppose p and q are statements. Then the logical disjunction $p \vee q$ (read "p or q") is the compound statement having the following truth table:

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- The disjunction ("or") of two statements is false only when both statements are false.
- Note the similarity of the logical disjunction operator $\vee$ to the set union operator $\cup$. They are closely related in meaning, also.

Inclusive OR versus Exclusive OR

The English word “or” is used in two different ways. Sometimes it is used to mean “p or q or both”, i.e. at least one of the two alternatives occurs, as above. But sometimes it is used to mean “p or q but not both”, i.e. exactly one of the two alternatives occurs. For example, “He will go to college or join the Army” uses “or” in the latter sense, called the exclusive or or exclusive disjunction. We will always use the first definition (inclusive) so that $p \vee q$ will always mean “p and/or q”, consistent with our truth table definition.

Statement Examples

Let p be "It is cold" and q be "It is raining." Translate the following logical expressions to English sentences:

1. $\sim p$
2. $p \wedge q$
3. $p \vee q$
4. $q \vee \sim p$

Let p be "He is tall" and let q be "He is handsome." Write each of the following in symbolic form using p and q:

1. He is tall and handsome.
2. He is tall but not handsome.
3. It is false that he is either short or handsome.

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1. He is tall and handsome. $p \wedge q$
2. He is tall but not handsome. $p \wedge \sim q$
3. It is false that he is either short or handsome. $\sim(\sim p \vee q)$

Logical Implication

Consider the statement "If it is raining then the street is wet." When is this statement true? If it is raining and the street is wet, then it is clearly true. Likewise, if it is raining and the street is dry, it is clearly false. What if it is not raining and the street is dry? This does not contradict our statement, so we still regard it as true. What if it is not raining, but the street is wet anyway? Well, a water pipe may have burst. We still regard the statement as true. Thus, the truth table:

Logical Implication

p	q	$p \implies q$
T	T	T
T	F	F
F	T	T
F	F	T

This is a statement of the form "If p, then q" (an implication) which is written mathematically as $p \implies q$ (read "p implies q").

Logical Negation

This concept is very simple. The logical negation of p is written $\sim p$ and has the opposite truth value as p. We read $\sim p$ as "not p"

Negation

p	$\sim p$
T	F
F	T

Examples:

p : Jack is a dog. $\sim p =$ Jack is not a dog.

$q : 3 > 2$ $\sim q : 3 \leq 2$

r : All cows are black. $\sim r$: It is not true that all cows are black (or Not all cows are black).

Note that for any statement p, exactly one of p and $\sim p$ is true and the other is false.

(Logical) Propositions and Truth Tables

Let p, q, r, ... be logical variables, that is, they represent statements which can be true or false. A logical statement built up from logical operators involving our logical variables is called a proposition.

Example (The Proposition $\sim(p \wedge \sim q)$)

The truth table for this propagation can be constructed as follows:

p	q	$\sim q$	$p \wedge \sim q$	$\sim(p \wedge \sim q)$
T	T			
T	F			
F	T			
F	F			

Note that since there are 2 variables (p and q), and each can take 2 values (T and F), the truth table requires $2^2 = 4$ rows. For a proposition involving n variables, 2^n rows would be needed for all possible combinations of inputs.

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(Logical) Propositions and Truth Tables (Cont.)

The truth table of the proposition consists precisely of the columns under the variables and the column under the proposition:

p	q	$\sim(p \wedge \sim q)$
T	T	T
T	F	F
F	T	T
F	F	T

The other columns were merely used to help construct this, the truth table of the proposition.

Tautologies and Contradictions

A proposition that is always true is called a tautology. A proposition that is always false is called a contradiction. As usual, we check the truth value of a proposition by making a truth table

Example ($p \vee \sim p$)

p	$\sim p$	$p \vee \sim p$
T	F	
F		

Example ($p \wedge \sim p$)

p	$\sim p$	$p \wedge \sim p$
T		
F		

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Example ($p \wedge \sim p$)

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

$p \wedge \sim p$ is a contradiction (always false).

Contrapositive

Fill in the following truth table:

p	q	$p \implies q$	$\sim q$	$\sim p$	$\sim q \implies \sim p$
T	T				
T	F				
F	T				
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Fill in the following truth table:

p	q	$p \implies q$	$\sim q$	$\sim p$	$\sim q \implies \sim p$
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Notice that $p \implies q$ and $\sim q \implies \sim p$ (the contrapositive statement) always take the same truth value, whatever the values of p and q. We say that such statements are equivalent and we write $(p \implies q) \equiv (\sim q \implies \sim p)$.

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Some examples of contrapositives:

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Some examples of contrapositives:

Statement: If it is raining, then the ground is wet.

Contrapositive

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Some examples of contrapositives:

Statement: If it is raining, then the ground is wet. **Contrapositive**: If the ground is not wet, then it is not raining.

Contrapositive

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Some examples of contrapositives:

Statement: If it is raining, then the ground is wet. **Contrapositive**: If the ground is not wet, then it is not raining.

Statement: If Joe hits above .350, then he will be the league MVP.

Contrapositive

Fill in the following truth table:

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Some examples of contrapositives:

Statement: If it is raining, then the ground is wet. **Contrapositive**: If the ground is not wet, then it is not raining.

Statement: If Joe hits above .350, then he will be the league MVP. **Contrapositive**: If Joe is not the league MVP, then he did not hit above .350.

Converse

The statement $q \implies p$ is called the converse of the statement $p \implies q$. Is the converse equivalent to the original proposition? Find out by filling in the truth table for the two propositions:

p	q	$p \implies q$	$q \implies p$
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A statement and its converse are not equivalent (in general)! Consider a (true, for the sake of discussion) campaign pledge made by a candidate for office: "If I am elected, taxes will be lower!" Suppose that taxes do go down. Can we conclude that the candidate was elected? What can we conclude if taxes do not go down? Why?

Laws of the Algebra of Propositions

Idempotent Laws

$p \vee p \equiv p$ $\quad$ $p \wedge p \equiv p$

Associative Laws

$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $\quad$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Commutative Laws

$p \vee q \equiv q \vee p$ $\quad$ $p \wedge q \equiv q \wedge p$

Distributive Laws

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $\quad$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Identity Laws

$p \vee \mathrm{F} \equiv p$ $\quad$ $p \wedge \mathrm{T} \equiv p$

$p \vee \mathrm{T} \equiv \mathrm{T}$ $\quad$ $p \wedge \mathrm{F} \equiv \mathrm{F}$

Laws of the Algebra of Propositions (Cont.)

Complement Laws

$$p \vee \sim p \equiv \mathrm{T} \qquad p \wedge \sim p \equiv \mathrm{F}$$

$$\sim \mathrm{T} \equiv \mathrm{F} \qquad \sim \mathrm{F} \equiv \mathrm{T}$$

Involution Law

$$\sim\sim p \equiv p$$

DeMorgan's Laws

$$\sim(p \vee q) \equiv \sim p \wedge \sim q \qquad \sim(p \wedge q) \equiv \sim p \vee \sim q$$