SEQUENCE AND SERIES WORKSHEET SOLUTIONS

PROBLEM 1

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- In any arithmetic sequence, the average is the median.
- Median = 2646 / 27 = 98.
- There are 13 numbers above the median.
- Answer. 98 + 13 = 111

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- $\Delta^2 = a\Delta$
- $\Delta = a$, so the sequence is a, 2a, 3a, 4a, ….
- Answer: $a/d = 1/4$.

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- Answer: Sum is 9(25) = 225.

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PROBLEM 4

- For all positive integers n less than 2002, let
  - an = 11, if n is divisible by 13 and 14;
  - an = 13, if n is divisible by 14 and 11;
  - an = 14, if n is divisible by 11 and 13;
  - an = 0, otherwise.
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- There are ten 11s, twelve 13s, and thirteen 14s.
- The sum is 10(11) + 12(13) + 13(14)
- Answer 448.

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- $a_m = 1 + 2 + \ldots + m$
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- $a_{12} = 12(13)/2 = 78$

PROBLEM 6

- Let $a_1, a_2, \ldots$ be a sequence for which $a_1 = 2$, $a_2 = 3$, and $a_n = a_{n-1}/a_{n-2}$ for each positive integer n > 2. What is a_{2006}?

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- so $a_{2006} = a_2 = 3$.

PROBLEM 7

- Consider the sequence of numbers: 4, 7, 1, 8, 9, 7, 6, ... For $n > 2$, the n^{th} term of the sequence is the units digit of the sum of the two previous terms. Let S_n denote the sum of the first n terms of this sequence. What is the smallest value of n for which $S_n > 10,000$?

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- $k = [10,000/60] = 166$, and $S_{12(166)} = 60(166) = 9960$.

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- This can be done by adding the next 7 terms of the sequence, since their sum is 42.

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- To have $S_n > 10,000$, add enough additional terms for their sum to exceed 40.
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- Thus, the smallest value of n is $12(166) + 7 = 1999$.

PROBLEM 8

- Suppose that $\{a_n\}$ is an arithmetic sequence with
- $a_1 + a_2 + \cdot \cdot \cdot + a_{100} = 100$ and
- $a_{101} + a_{102} + \cdot \cdot \cdot + a_{200} = 200$.
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- Let $\Delta = a_2 - a_1$
- $a_{101} - a_1 = 100\Delta = a_{100+n} - a_n$ for all n
- $(a_{101} + a_{102} + \cdot \quad \cdot \quad \cdot \quad + a_{200}) = 200$
- $-(a_1 \quad + a_2 \quad + \cdot \quad \cdot \quad \cdot \quad + a_{100}) = 100$

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- $100(100\Delta) = 100$

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- $100(100\Delta) = 100$
- $\Delta = 1/100 = a_2 - a_1$

PROBLEM 9

- Given a finite sequence $S = (a_1, a_2, \ldots, a_n)$ of n real numbers, let $A(S)$ be the sequence $((a_1 + a_2)/2, (a_2 + a_3)/2, \ldots, (a_{n-1} + a_n)/2)$ of n-1 real numbers. Define $A^1(S) = A(S)$ and, for each integer m, $1 < m < n$, define $A^m(S) = A(A^{m-1}(S))$. Suppose $x > 0$, and let $S = (1, x, x^2, \ldots, x^{100})$. If $A^{100}(S) = 1/2^{50}$, then what is x ?

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- It is clear that the denominator of each term of $A^m(S) = 2^m$.
- Let's investigate the numerators of each term.

PROBLEM 9

- S is $1, x, x^2, x^3, x^4, \ldots$

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- S is 1, x, x^2, x^3, x^4, …
- The numerators of $A^1(S)$ = $1+x$, $x+x^2$, x^2+x^3, …

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- The numerators of $A^3(S)$ = $1+3x+3x^2+x^3$, …

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- The numerator of the 1st term of $A^m(S)$ = $(1+x)^m$

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- The numerator of the 1st term of $A^m(S)$ = $(1+x)^m$
- $A^m(S)$ has one fewer term than $A^{m-1}(S)$ and $A^1(S)$ has 100 terms

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- The numerator of the 1st term of $A^m(S) = (1+x)^m$
- $A^m(S)$ has one fewer term than $A^{m-1}(S)$ and $A^1(S)$ has 100 terms
- $A^{100}(S)$ has exactly one term.

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- S is 1, x, x^2, x^3, x^4, …
- The numerators of $A^1(S) = 1+x,\ x+x^2,\ x^2+x^3,\ \ldots$
- The numerators of $A^2(S) = 1+2x+x^2,\ x+2x^2+x^3,\ldots$
- The numerators of $A^3(S) = 1+3x+3x^2+x^3,\ \ldots$
- The numerator of the 1st term of $A^m(S) = (1+x)^m$
- $A^m(S)$ has one fewer term than $A^{m-1}(S)$ and $A^1(S)$ has 100 terms
- $A^{100}(S)$ has exactly one term.
- $A^{100}(S) = (1 + x)^{100} / 2^{100} = 1 / 2^{50}$

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- $(1 + x)^{100} / 2^{100} = 1 / 2^{50}$

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- $(1 + x)^{100} / 2^{100} = 1 / 2^{50}$
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- $(1 + x)^2 = 2$

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- $(1 + x)^{100} / 2^{100} = 1 / 2^{50}$
- $(1 + x)^2 / 2^2 = 1 / 2$
- $(1 + x)^2 = 2$
- $1 + x = \sqrt{2}$

PROBLEM 9

- $(1 + x)^{100} / 2^{100} = 1 / 2^{50}$
- $(1 + x)^2 / 2^2 = 1 / 2$
- $(1 + x)^2 = 2$
- $1 + x = \sqrt{2}$
- $x = \sqrt{2} - 1$