NUMBER THEORY WORKSHEET SOLUTIONS

PROBLEM 1

- Let n be a positive integer such that $1/2 + 1/3 + 1/7 + 1/n$ is an integer. Which of the following statements is not true:
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- The LCD of 1/2, 1/3, 1/7 is $2(3)(7) = 42$
- n must be 42 or a divisor of 42
- so $n > 84$ is false.
- Answer: E

PROBLEM 2

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- for $n = 1$, $m(1) < m + 1$
- True for all positive integers m
- Answer: Infinite

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- To be prime, one factor = 1, the other factor is a prime
- The only value of n that works is 3
- Answer: 1

PROBLEM 4

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- smallest sum is 3(4), largest is 3(16),
- all intermediate values are possible.
- Answer: 13

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- $2^{11} = 2048$
- *Answer: 14*

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  What is A + M + C?
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- *So 2(AMC) = 1234, the first 4 digits of the sum*
- *AMC = 617*
- *Answer: 14*

PROBLEM 7

- *The Contest Chair noticed that his airport parking receipt had digits of the form bbcac,where $0 \le a < b < c \le 9$, and b was the average of a and c. How many 5-digit numbers satisfy all these properties?*

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- *Looking at the even values: a = 0 has 4 possible values of c, a = 2 has 3, a= 4 has 2, a = 6 has 1*
- *There are a total of 10 even possible values of a and c.*
- *Similarly, there are 10 odd possible values of a and c.*
- *Answer: 20*

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- Each even counting number is one more than the corresponding odd counting number (e.g., 2 and 1, 4 and 3, 6 and 5).
- There are 2003 pairs of numbers, each with a difference of 1.
- Answer: 2003

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- Then e cannot be 7, 5, or 2 because 77, 75, 72 are not prime.

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- e = 3 works because 73 is prime.
- 7(3)(73) = 1533
- Answer: 12