2009 Fall Startup Event
Thursday, September 24th, 2009

This test consists of 100 problems to be solved in 30 minutes. All answers must be exact, complete, and in simplest form. **To ensure consistent grading, if you get a decimal, mixed number, or ratio as any part of an answer, it should be expressed as a fraction unless otherwise specified in the problem.** A correct answer to a problem scores one point; a blank or incorrect answer to a problem scores no points. All answers must be written on the answer sheet in the boxes provided; work or answers written elsewhere will not be scored.

1. Evaluate: $\begin{array}{r} 352 \\ \underline{+184} \end{array}$

2. Evaluate: $8\overline{)552}$

3. Evaluate: $\frac{5}{6} \times \frac{10}{21}$

4. 48 is what percent of 160?

5. How many centimeters are in three-and-a-half meters?

6. Evaluate: $2(4-6)^3+8$

7. Express in simplest radical form: $\sqrt{252}$

8. Simplify by rationalizing the denominator and reducing if necessary: $\frac{84}{4+\sqrt{2}}$

9. Evaluate: $19^2 - 15^2$

10. What digit is in the tenths place in the decimal number 987.654?

11. What value(s) of *z* satisfy $6z+11=113$?

12. What value(s) of *y* satisfy $4(y+7)-5=3(2y-9)$?

13. What value(s) of *x* satisfy $3x^2-14x-24=0$?

14. Express the solution to the system of equations $3w+v=14$ and $w-2v=7$ as an ordered pair in the form (w,v).

15. If it takes James three hours to do the dishes and John can do them in two hours, how many **minutes** will it take the two of them to do the dishes working together?

16. How many hours will it take Cherie to drive 1200 miles to Los Angeles at an average speed of 48 miles per hour?

17. If the sum of two numbers is 87 and their difference is 19, what is the value of the smaller number?

18. Express the equation of the line through the points $(1,-7)$ and $(-2,-1)$ in slope-intercept $(y = mx + b)$ form.

19. What is the slope of a line perpendicular to the line $6x + 4y = 21$?

20. What is the minimum distance from the point $(4,-6)$ to the line $4x - 3y = 5$?

21. Write the equation of a line in slope-intercept form that will never intersect the line $6x - 3y = 11$.

22. What are the coordinates, as an ordered pair in the form (x, y), of the vertex of the parabola with equation $y = 4x^2 - 48x + 17$?

23. If you can buy 4 forks with N nickels, how many dimes would be needed to buy F forks?

24. My piggy bank contains 42 coins with a total value of $4.00. If my piggy bank contains only nickels and dimes, how many nickels are in the piggy bank?

25. How many minutes is it from quarter-to-three to (the next) ten-past-six?

26. What value(s) of *u* satisfy $\frac{u+21}{u-9} = \frac{u+15}{u-7}$?

27. What is the coefficient of the t^2 term in the expansion of $(2t-3)(t+2)(3t+4)$?

28. In which quadrant does the point $(-3,8)$ lie?

29. What is the distance between the *x*- and *y*-intercepts of the line $2x + 4y = 20$?

30. What are the coordinates, as an ordered pair in the form (x, y), of the reflection of the point $(5,17)$ across the line $y = -x$?

31. A right triangle has a hypotenuse measuring 8 cm and a leg measuring 6 cm. What is the length, in centimeters, of the other leg?

32. What is the area, in square centimeters, of an equilateral triangle with sides measuring 10 cm each?

33. What is the name for a triangle with exactly two congruent sides?

34. If two sides of a triangle measure 37 and 73 cm, how many integer numbers of centimeters are possible for the length of the third side?

35. What is the perimeter, in centimeters, of a rectangle with sides measuring 14 and 11 cm?

36. What is the area, in square centimeters, of a $60°$ sector of a circle with a diameter of 12 cm?

37. What is the perimeter, in centimeters, of a regular pentagon with sides measuring 22 cm each?

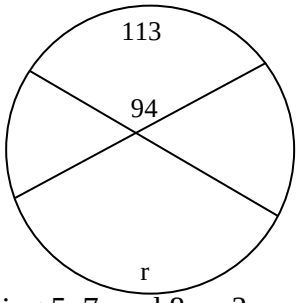
38. What is the measure, in degrees, of an interior angle of a regular 15-gon?

39. If a quadrilateral with sides of 3, 4, 5, and 6 cm and an area of 2 cm^2 is similar to one with two sides of 12 and 24 cm, what is the area, in square centimeters, of the larger quadrilateral?

40. What is the volume, in cubic centimeters, of a right circular cone with a base radius of 9 cm and a height of 6 cm?

41. How many vertices does a regular octahedron have?

42. In the figure shown with all angles measured in degrees, what is the value of *r*?



43. What is the area, in square centimeters, of a triangle with sides measuring 5, 7, and 8 cm?

44. An equilateral triangle is inscribed in a circle with a 24 cm diameter. What is the area, in square centimeters, of the largest circle that can be "inscribed" externally tangent to the triangle but internally tangent to the original circle?

45. Two circles with radii of 12 and 18 cm have their centers 36 cm apart. What is the length, in centimeters, of one of their common internal tangents?

46. In a four by seven array of unit squares, how many squares of any size are there?

47. How many diagonals can be drawn in a convex 16-gon?

48. When three triangles are drawn in the same plane, what is the largest number of regions into which they can divide the plane?

49. What is the area, in square centimeters, of a square circumscribed about a circle with a radius of 8 cm?

50. What is the surface area, in square centimeters, of a right rectangular prism with edges measuring 9, 6, and 13 cm?

51. Evaluate: $(3-5i)(i+2)$

52. What are the coordinates, as an ordered pair in the form (x, y), of the center of the conic section with equation $2x^2 - 5y^2 + 8x + 30y = 101$.

53. In how many points do the graphs of $y = \frac{1}{x+2}$ and $y = 5 - |x - 3|$ intersect?

54. What is the smallest integer value of q satisfying $3 \cdot 2^q + 2^{2q} > 1000$?

55. If $n(p) = \frac{1}{3}p + 50$, evaluate $n^{-1}(41)$.

56. Express the range of the function $k(m) = \frac{3m+6}{|m+1|+1}$ in interval notation if the domain and range must both be real numbers.

57. If I invest $3000 in a bank account that pays 5% annual interest compounded continuously without rounding, exactly how much money, in dollars, will I have at the end of 20 years?

58. What is the product of the roots of $3j^3 + 4j^2 - 5j + 6 = 0$?

59. What is the coefficient of the h^3 term when $(2h-1)^5$ is expanded and like terms are combined?

60. Evaluate: $\log_{16} 512$

61. Convert the base 10 number 235_{10} to a base six number.

62. What is the prime factorization, in exponential form, of 840?

63. What is the sum of the positive integer factors of 108?

64. What is the least common multiple of 135 and 72?

65. If the six-digit number 82G459 is divisible by 9, what is the value of the digit G?

66. What is the smallest prime number greater than 53?

67. When two fair six-sided dice are rolled, what is the probability that the sum of the numbers on their upper faces is ten?

68. When three tiles are drawn from a bag containing three blue tiles and five red tiles, what is the probability that two blues and a red are drawn?

69. In how many ways can five identical green coins and three identical white coins be placed relative to one another at eight equally spaced identical positions about a round table?

70. Of 94 students surveyed, 73 like Mr. Norris and 37 like both Mr. Norris and Mr. Brown. If only one student likes neither, how many students like Mr. Brown?

71. Tom and Katie take turns rolling a single fair six-sided die, with Katie going first. If Katie rolls a 4 or less, she wins and the game is done. If Tom rolls a 3 or less, he wins and the game is done. What is the probability that Tom wins?

72. In a dull game of chance, you pay five dollars to draw a single card from a standard 52-card deck. If the card is black, you receive 7 dollars, otherwise you receive nothing. What is your expected gain, in dollars rounded to the nearest hundredth (cent) when you play this game (it could be negative if you will lose money on average)?

73. What is the fifth term of the geometric sequence with first term of 9 and common ratio of 5?

74. What is the next term of the sequence beginning 10, 12, 11, 15, 13, 18, 16, 21, 20, ___?

75. Evaluate: $\sum_{f=2}^{7} \frac{1}{f^2-1}$

76. What is the sum of the first 12 terms of the arithmetic sequence with first term 21 and common difference –4?

77. What is the sum of the 17 smallest positive odd numbers?

78. What is the sum of the 13 smallest positive perfect squares?

79. When their class lines up for recess, Annie is by David, Eddie is ahead of Cindy, and Ben is directly behind Eddie. If David is directly in front of Cindy, who is in the center of the line?

80. In the cryptarithm shown, where each letter represents a different digit 0-9, the same letter always represents the same digit, and two different letters always represent two different digits, what is the largest possible value of the three-digit number ABC?

$$\begin{array}{r} AB \\ +AB \\ \hline BC \end{array}$$

81. Evaluate: $\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}$

82. Evaluate: $1+2+3-4+5+6+7-8+\cdots+97+98+99-100$

83. Using the numerals 9, 8, and 6 exactly once each and the operations +, −, ×, and ÷ (and parentheses) as often as you like, write an expression that evaluates to 12.

84. What is the median of the data set {4, 7, 11, 8, 11}?

85. In a five-element data set of integer test scores from 0 to 100 inclusive, the mean is 48 and the single mode is 60. What is the lowest possible value of the median?

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86. Evaluate: $\begin{bmatrix}1 & -2 & 3\end{bmatrix}\begin{bmatrix}-2 & 1\\ 0 & -1\\ 2 & -3\end{bmatrix}$

87. What is the area, in square units, of the triangle in the Cartesian Plane with vertices at the points $(1,5)$, $(-3,2)$, and $(4,-1)$?

88. If set D is the set of all multiples of 5 between 24 and 89 and set E is the set of all multiples of 3 between 29 and 94, how many elements are in the set $D \cap E'$?

89. If F and G are angles in the first quadrant and $\sin F = \frac{12}{13}$ and $\sin G = \frac{3}{5}$, evaluate $\sin(F+G)$.

90. Evaluate: $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

91. Express the point with spherical coordinates $\left(8, \frac{\pi}{6}, \frac{\pi}{4}\right)$ in cylindrical coordinates.

92. In a triangle with sides measuring 5, 12, and 13 cm, what is the tangent of the smallest angle?

93. Convert $100°$ to radians.

94. In ΔHIJ, $IJ = 12$, $HJ = 18$, and $J = 120°$. What is the area of ΔHIJ?

95. If $k(m) = (2m-3)^4$, evaluate $k'(0)$.

96. A water tank in the shape of an inverted cone has a top radius of 8 m and a height of 4 m. If the water is draining out at a constant rate of 100 m^3/s, how fast is the water level dropping (in m/s) when the depth is 3 m?

97. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 3\sin(2x)\,dx$

98. What is the area of the closed region bounded by the x-axis and the graph of $y = 16 - x^4$?

99. What is the equation, in slope-intercept form, of the line tangent to the graph of $y = x^3$ at the point where $x = -3$?

100. What is the average value of the function $y = x^2 - 2$ between $x = 1$ and $x = 5$?