Contest 1

October 20, 2009

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We are looking for one value, so $x = 0$ is it.

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$1 + $2 = $3 and all other amounts are > $4, so $4 is one value.

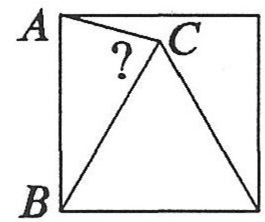
The total of the five coupons is $24.

There is also no way to take $4 away from all of the coupons.

$20 is the other value.

Problem 3

An Equilateral triangle and a square share a common side, as shown. In ΔABC, what is $m\angle ACB$?



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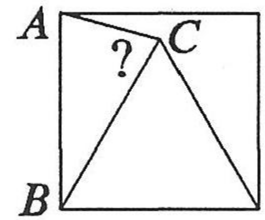

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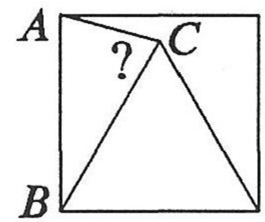

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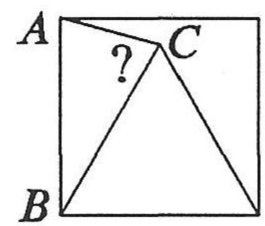

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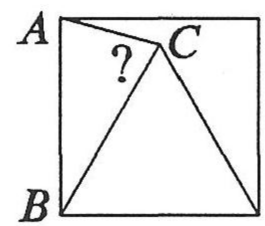

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$m\angle ACB = (180\text{-}30)/2 = 75$

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Answer: (1, 4, 7)

Note (2, 1, 9) also works – replace 3 30s with 2 31s + 1 28.

Problem 6

Last year, each of Big Al’s 5 brothers gave a gift of money to Big Al. The dollar amounts were consecutive integers, and their sum was a perfect cube. If the brothers give Big Al cash gifts with those same properties both this year and next year as well, but this year’s sum is larger than last year’s, and next year’s sum is larger still, what is the least possible dollar amount Big Al could get next year from his 5 brothers combined?

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The total gifts next year is $5(25)(3)^3 = \$3375$

Problem 5

In a certain two person game (one of many varieties of NIM), each person, in turn, removes 1, 2, 3, 4, or 5 toothpicks from a common pile until the pile is exhausted. The person who takes the last toothpick loses. If the starting pile contains 300 toothpicks, how many toothpicks must the first player take on the first turn to guarantee a win with perfect subsequent play?

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The largest number < 300 of that form is 295. Take 5.

Problem 5A

We make two changes to the rules of the NIM game.

1) The person that takes the last toothpick WINS.

2) Each turn, the person may take as many toothpicks as he/she wishes, but not more than was taken on the other person's last turn. To begin, the first person may take any number except all of the toothpicks.

How many toothpicks must the first person take to guarantee a win with perfect play? How does this vary with the initial number of toothpicks?