Contest 2

November 17, 2009

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The bottle itself weighs 800 – 650 = 150 g.

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EXTRA TIDBIT

$629 = 25^2 + 2^2 = 23^2 + 10^2$

Any number that can be written as the sum of two squares in two different ways is always composite.

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$x = 2, -1$

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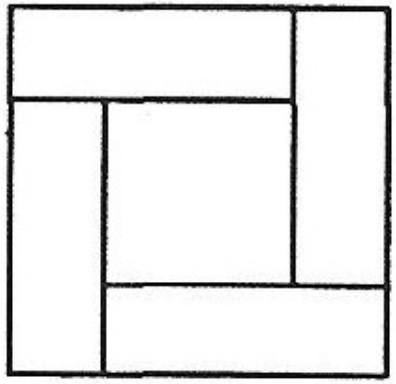
In the diagram at the right, a square of area 2009 is divided into a small square and four congruent rectangles. What is the perimeter of one of the four congruent rectangles?



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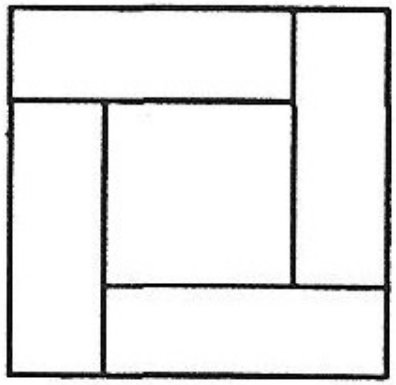

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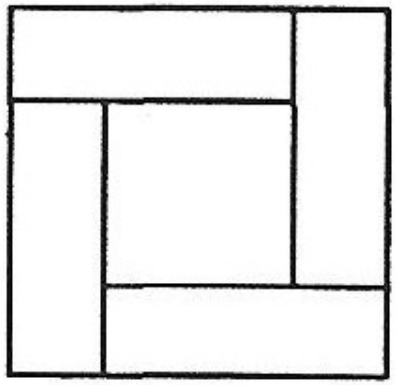

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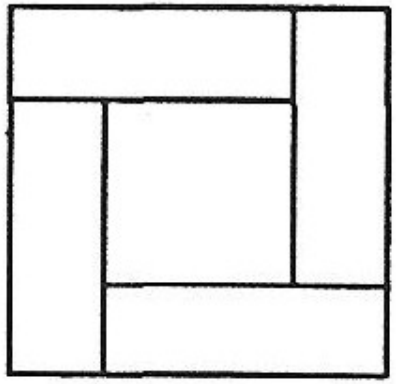

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$\text{Perim} = 2(L+W) = 2\sqrt{2009} = 14\sqrt{41}$

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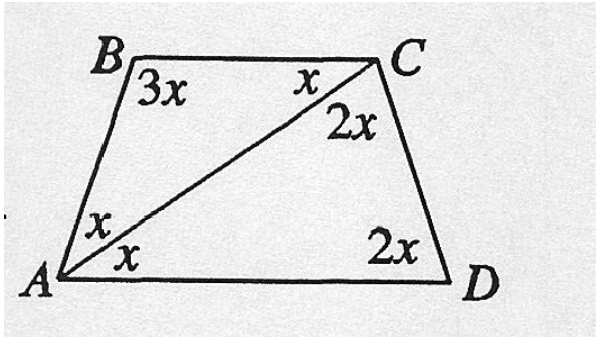

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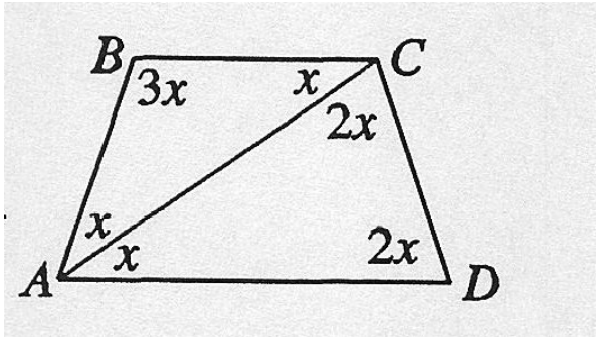

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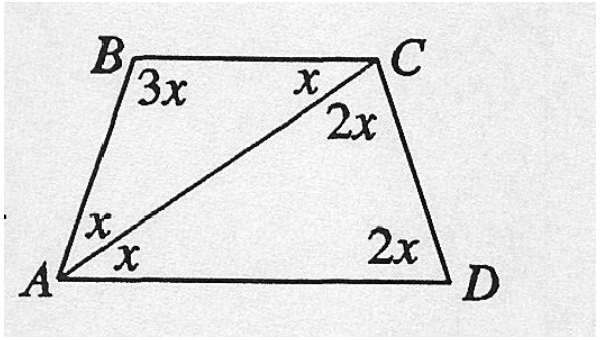

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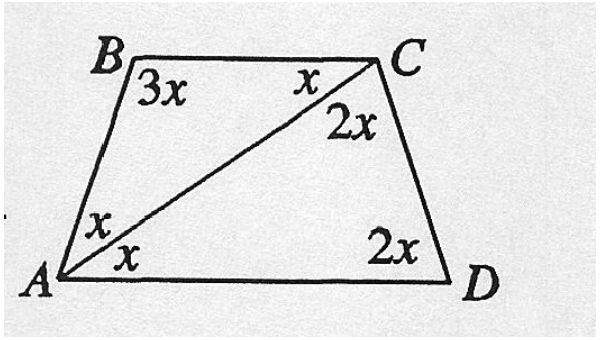

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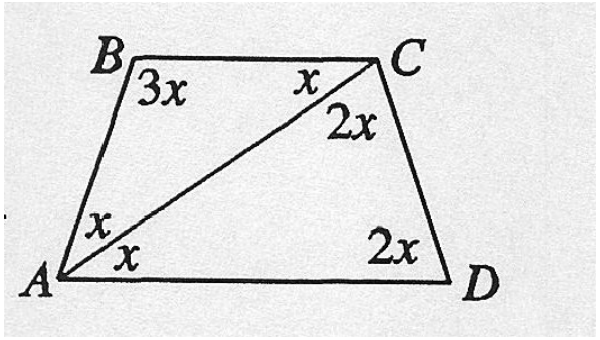

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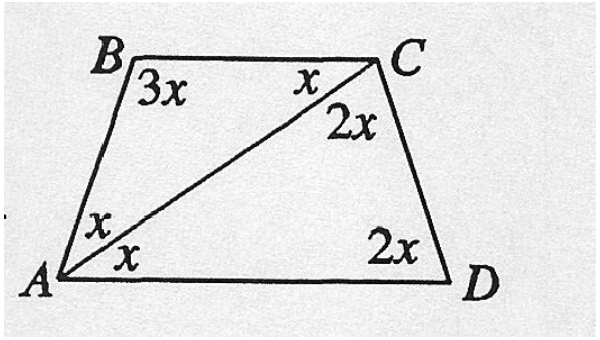

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m∠*ADC* = 2*x* = 72

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Al and Bo, who together have $168, bet against each other. Each bets the same fraction of his money as the other bets. If Al wins, he'll have double what Bo then has. If Bo wins, he'll have triple what Al then has. How many dollars does Al have at the start?

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$A + 4A/3 = 168$

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$(4A-3B)(A+B) = 0 \Rightarrow 4A-3B = 0$
$B = 4A/3$
$A + 4A/3 = 168$
$A = 3(168)/7 = 72$